Chapter 6

Circular Motion and Other Applications of Newton's Laws

▲ *The London Eye, a ride on the River Thames in downtown London. Riders travel in a large vertical circle for a breathtaking view of the city. In this chapter, we will study the forces involved in circular motion. (© Paul Hardy/CORBIS)*

In the preceding chapter we introduced Newton's laws of motion and applied them to situations involving linear motion. Now we discuss motion that is slightly more complicated. For example, we shall apply Newton's laws to objects traveling in circular paths. Also, we shall discuss motion observed from an accelerating frame of reference and motion of an object through a viscous medium. For the most part, this chapter consists of a series of examples selected to illustrate the application of Newton's laws to a wide variety of circumstances.

6.1 Newton's Second Law Applied to Uniform Circular Motion

In Section 4.4 we found that a particle moving with uniform speed v in a circular path of radius r experiences an acceleration that has a magnitude

$$a_c = \frac{v^2}{r}$$

The acceleration is called *centripetal acceleration* because $\mathbf{a}_c$ is directed toward the center of the circle. Furthermore, $\mathbf{a}_c$ is *always* perpendicular to $\mathbf{v}$. (If there were a component of acceleration parallel to $\mathbf{v}$, the particle's speed would be changing.)

Consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path, as illustrated in Figure 6.1. Its weight is supported by a frictionless table. Why does the ball move in a circle? According to Newton's first law, the ball tends to move in a straight line; however, the string prevents

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An athlete in the process of throwing the hammer at the 1996 Olympic Games in Atlanta, Georgia. The force exerted by the chain causes the centripetal acceleration of the hammer. Only when the athlete releases the hammer will it move along a straight-line path tangent to the circle.

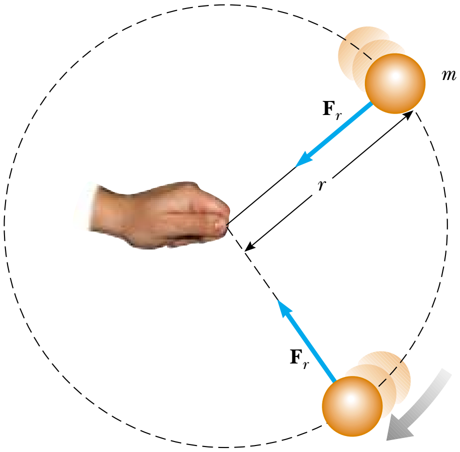


Figure 6.1 Overhead view of a ball moving in a circular path in a horizontal plane. A force $\mathbf{F}_r$ directed toward the center of the circle keeps the ball moving in its circular path.

motion along a straight line by exerting on the ball a radial force $\mathbf{F}_r$ that makes it follow the circular path. This force is directed along the string toward the center of the circle, as shown in Figure 6.1.

If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

Force causing centripetal acceleration

$$\sum F = ma_c = m\frac{v^2}{r} \tag{6.1}$$

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle. This idea is illustrated in Figure 6.2 for the ball whirling at the end of a string in a horizontal plane. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string breaks.

Figure 6.3 (Quick Quiz 6.1 and 6.2) A Ferris wheel located on the Navy Pier in Chicago, Illinois.

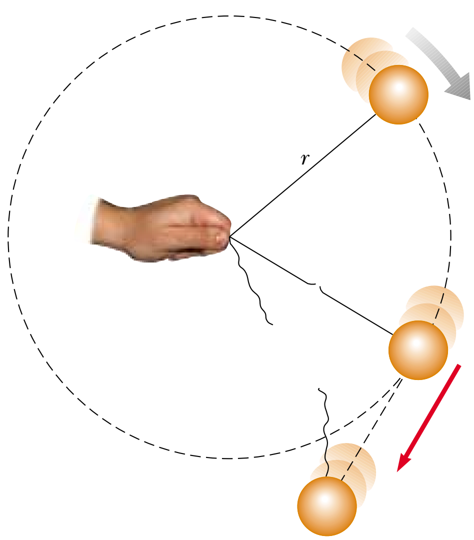
Quick Quiz 6.1 You are riding on a Ferris wheel (Fig. 6.3) that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation—it does not invert. What is the direction of your centripetal acceleration when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of your centripetal acceleration when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

Quick Quiz 6.2 You are riding on the Ferris wheel of Quick Quiz 6.1. What is the direction of the normal force exerted by the seat on you when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of the normal force exerted by the seat on you when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

PITFALL PREVENTION

6.1 Direction of Travel When the String is Cut

Study Figure 6.2 very carefully. Many students (wrongly) think that the ball will move *radially* away from the center of the circle when the string is cut. The velocity of the ball is *tangent* to the circle. By Newton's first law, the ball continues to move in the direction that it is moving just as the force from the string disappears.



Active Figure 6.2 An overhead view of a ball moving in a circular path in a horizontal plane. When the string breaks, the ball moves in the direction tangent to the circle.

At the Active Figures link at http://www.pse6.com, you can "break" the string yourself and observe the effect on the ball's motion.

Conceptual Example 6.1 Forces That Cause Centripetal Acceleration

The force causing centripetal acceleration is sometimes called a *centripetal force*. We are familiar with a variety of forces in nature—friction, gravity, normal forces, tension, and so forth. Should we add *centripetal* force to this list?

Solution No; centripetal force *should not* be added to this list. This is a pitfall for many students. Giving the force causing circular motion a name—centripetal force—leads many students to consider it as a new *kind* of force rather than a new *role* for force. A common mistake in force diagrams is to draw all the usual forces and then to add another vector for the centripetal force. But it is not a separate force—it is simply one or more of our familiar forces *acting in the role of a force that causes a circular motion.*

Consider some examples. For the motion of the Earth around the Sun, the centripetal force is *gravity*. For an object sitting on a rotating turntable, the centripetal force is *friction*. For a rock whirled horizontally on the end of a string, the magnitude of the centripetal force is the *tension* in the string. For an amusement-park patron pressed against the inner wall of a rapidly rotating circular room, the centripetal force is the *normal force* exerted by the wall. Furthermore, the centripetal force could be a combination of two or more forces. For example, as you pass through the lowest point of the Ferris wheel in Quick Quiz 6.1, the centripetal force on you is the difference between the normal force exerted by the seat and the gravitational force. We will not use the term *centripetal force* in this book after this discussion.

Example 6.2 The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure 6.4. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum.*) Find an expression for v.

Solution Conceptualize the problem with the help of Figure 6.4. We categorize this as a problem that combines equilibrium for the ball in the vertical direction with uniform circular motion in the horizontal direction. To analyze the problem, begin by letting θ represent the angle between the string and the vertical. In the free-body diagram shown, the force $\mathbf{T}$ exerted by the string is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of revolution. Because the object does not accelerate in the vertical direction, $\sum F_y = ma_y = 0$ and the upward vertical component of $\mathbf{T}$ must balance the downward gravitational force. Therefore,

$$(1) \qquad T\cos\theta = mg$$

Because the force providing the centripetal acceleration in this example is the component $T\sin\theta$, we can use Equation 6.1 to obtain

$$(2) \qquad \sum F = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Dividing (2) by (1) and using $\sin\theta/\cos\theta = \tan\theta$, we eliminate T and find that

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

From the geometry in Figure 6.4, we see that $r = L\sin\theta$; therefore,

$$\boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

Note that the speed is independent of the mass of the object.

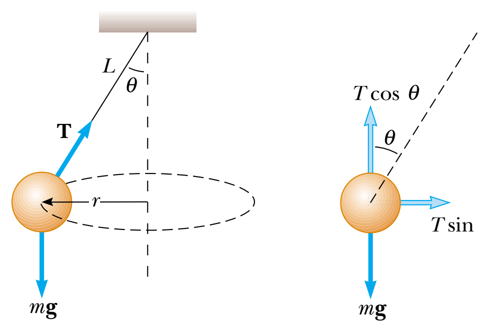


Figure 6.4 (Example 6.2) The conical pendulum and its free-body diagram.

Example 6.3 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during the motion.

Solution It makes sense that the stronger the cord, the faster the ball can twirl before the cord breaks. Also, we expect a more massive ball to break the cord at a lower speed. (Imagine whirling a bowling ball on the cord!)

Because the force causing the centripetal acceleration in this case is the force $\mathbf{T}$ exerted by the cord on the ball, Equation 6.1 yields

$$(1) \qquad T = m\frac{v^2}{r}$$

Solving for v, we have

$$v = \sqrt{\frac{Tr}{m}}$$

This shows that v increases with T and decreases with larger m, as we expect to see—for a given v, a large mass requires a large tension and a small mass needs only a small tension. The maximum speed the ball can have corresponds to the maximum tension. Hence, we find

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0 \text{ N})(1.50 \text{ m})}{0.500 \text{ kg}}} = \boxed{12.2 \text{ m/s}}$$

What If? **Suppose that the ball is whirled in a circle of larger radius at the same speed *v*. Is the cord more likely to break or less likely?**

Answer The larger radius means that the change in the direction of the velocity vector will be smaller for a given time interval. Thus, the acceleration is smaller and the required force from the string is smaller. As a result, the string is less likely to break when the ball travels in a circle of larger radius. To understand this argument better, let us write Equation (1) twice, once for each radius:

$$T_1 = \frac{mv^2}{r_1} \qquad T_2 = \frac{mv^2}{r_2}$$

Dividing the two equations gives us,

$$\frac{T_2}{T_1} = \frac{\left(\dfrac{mv^2}{r_2}\right)}{\left(\dfrac{mv^2}{r_1}\right)} = \frac{r_1}{r_2}$$

If we choose $r_2 > r_1$, we see that $T_2 < T_1$. Thus, less tension is required to whirl the ball in the larger circle and the string is less likely to break.

Example 6.4 What Is the Maximum Speed of the Car?

Interactive

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure 6.5. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

Solution In this case, the force that enables the car to remain in its circular path is the force of static friction. (*Static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the road.) Hence, from Equation 6.1 we have

$$(1) \qquad f_s = m\frac{v^2}{r}$$

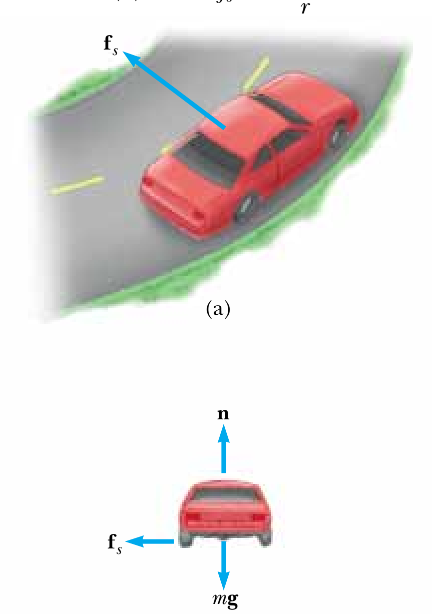


Figure 6.5 (Example 6.4) (a) The force of static friction directed toward the center of the curve keeps the car moving in a circular path. (b) The free-body diagram for the car.

The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\max} = \mu_s n$. Because the car shown in Figure 6.5b is in equilibrium in the vertical direction, the magnitude of the normal force equals the weight ($n = mg$) and thus $f_{s,\max} = \mu_s mg$. Substituting this value for f_s into (1), we find that the maximum speed is

$$(2) \qquad v_{\max} = \sqrt{\frac{f_{s,\max} r}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$

$$= \sqrt{(0.500)(9.80 \text{ m/s}^2)(35.0 \text{ m})}$$

$$= \boxed{13.1 \text{ m/s}}$$

Note that the maximum speed does not depend on the mass of the car. That is why curved highways do not need multiple speed limit signs to cover the various masses of vehicles using the road.

What If? **Suppose that a car travels this curve on a wet day and begins to skid on the curve when its speed reaches only 8.00 m/s. What can we say about the coefficient of static friction in this case?**

Answer The coefficient of friction between tires and a wet road should be smaller than that between tires and a dry road. This expectation is consistent with experience with driving, because a skid is more likely on a wet road than a dry road.

To check our suspicion, we can solve (2) for the coefficient of friction:

$$\mu_s = \frac{v_{\max}^2}{gr}$$

Substituting the numerical values,

$$\mu_s = \frac{v_{\max}^2}{gr} = \frac{(8.00 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 0.187$$

This is indeed smaller than the coefficient of 0.500 for the dry road.

Study the relationship between the car's speed, radius of the turn, and the coefficient of static friction between road and tires at the Interactive Worked Example link at **http://www.pse6.com.**

Example 6.5 The Banked Exit Ramp Interactive

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 50.0 m. At what angle should the curve be banked?

Solution On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between car and road, as we saw in the previous example. However, if the road is banked at an angle θ, as in Figure 6.6, the normal force $\mathbf{n}$ has a horizontal component $n \sin\theta$ pointing toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $n_x = n\sin\theta$ causes the centripetal acceleration. Hence, Newton's second law for the radial direction gives

$$(1) \qquad \sum F_r = n\sin\theta = \frac{mv^2}{r}$$

The car is in equilibrium in the vertical direction. Thus, from $\sum F_y = 0$ we have

$$(2) \qquad n\cos\theta = mg$$

Dividing (1) by (2) gives

$$(3) \qquad \tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{(13.4\text{ m/s})^2}{(50.0\text{ m})(9.80\text{ m/s}^2)}\right) = \boxed{20.1°}$$

If a car rounds the curve at a speed less than 13.4 m/s, friction is needed to keep it from sliding down the bank (to the left in Fig. 6.6). A driver who attempts to negotiate the curve at a speed greater than 13.4 m/s has to depend on friction to keep from sliding up the bank (to the right in Fig. 6.6). The banking angle is independent of the mass of the vehicle negotiating the curve.

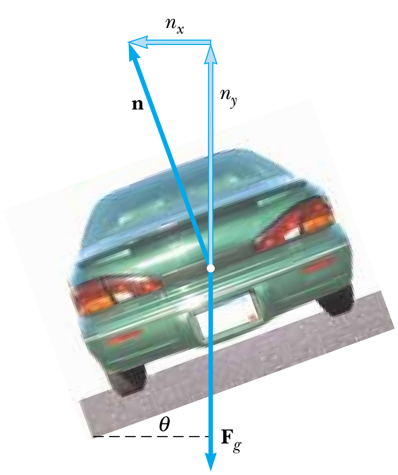


Figure 6.6 (Example 6.5) A car rounding a curve on a road banked at an angle θ to the horizontal. When friction is neglected, the force that causes the centripetal acceleration and keeps the car moving in its circular path is the horizontal component of the normal force.

What If? **What if this same roadway were built on Mars in the future to connect different colony centers; could it be traveled at the same speed?**

Answer The reduced gravitational force on Mars would mean that the car is not pressed so tightly to the roadway. The reduced normal force results in a smaller component of the normal force toward the center of the circle. This smaller component will not be sufficient to provide the centripetal acceleration associated with the original speed. The centripetal acceleration must be reduced, which can be done by reducing the speed v.

Equation (3) shows that the speed v is proportional to the square root of g for a roadway of fixed radius r banked at a fixed angle θ. Thus, if g is smaller, as it is on Mars, the speed v with which the roadway can be safely traveled is also smaller.

You can adjust the turn radius and banking angle at the Interactive Worked Example link at **http://www.pse6.com.**

Example 6.6 Let's Go Loop-the-Loop!

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in Figure 6.7a. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot **(A)** at the bottom of the loop and **(B)** at the top of the loop. Express your answers in terms of the weight of the pilot mg.

Solution To conceptualize this problem, look carefully at Figure 6.7. Based on experiences with driving over small hills in a roadway, or riding over the top of a Ferris wheel, you would expect to feel lighter at the top of the path. Similarly, you would expect to feel heavier at the bottom of the path. By looking at Figure 6.7, we expect the answer for (A) to be greater than that for (B) because at the bottom of the loop the normal and gravitational forces act in *opposite* directions, whereas at the top of the loop these two forces act in the *same* direction. The vector sum of these two forces gives the force of constant magnitude that keeps the pilot moving in a circular path at a constant speed. To yield net

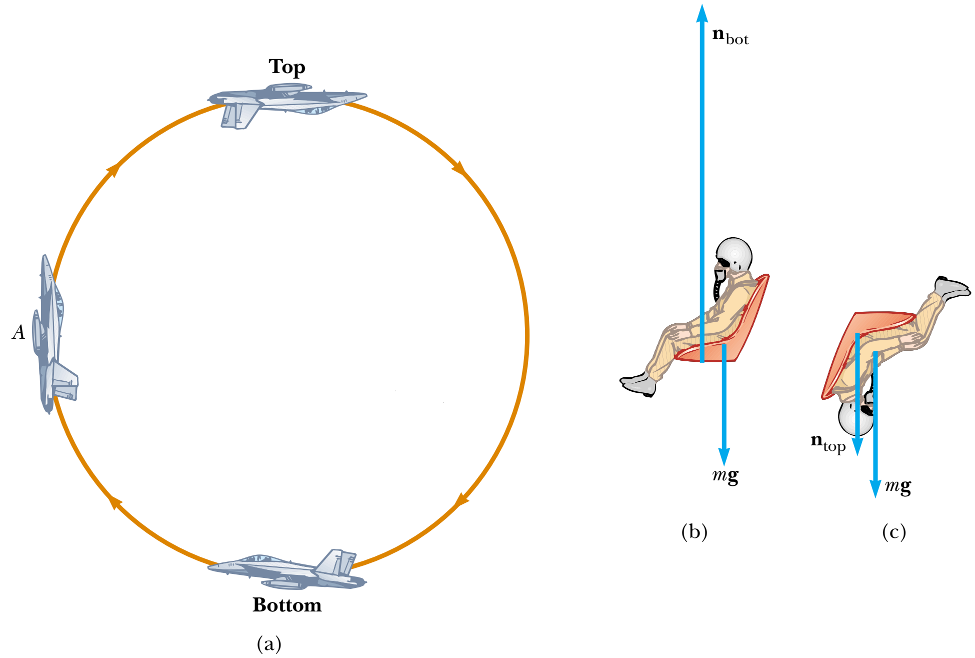


Figure 6.7 (Example 6.6) (a) An aircraft executes a loop-the-loop maneuver as it moves in a vertical circle at constant speed. (b) Free-body diagram for the pilot at the bottom of the loop. In this position the pilot experiences an apparent weight greater than his true weight. (c) Free-body diagram for the pilot at the top of the loop.

force vectors with the same magnitude, the normal force at the bottom must be greater than that at the top. Because the speed of the aircraft is constant (how likely is this?), we can categorize this as a uniform circular motion problem, complicated by the fact that the gravitational force acts at all times on the aircraft.

(A) Analyze the situation by drawing a free-body diagram for the pilot at the bottom of the loop, as shown in Figure 6.7b. The only forces acting on him are the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the upward force $\mathbf{n}_{\text{bot}}$ exerted by the seat. Because the net upward force that provides the centripetal acceleration has a magnitude $n_{\text{bot}} - mg$, Newton's second law for the radial direction gives

$$\sum F = n_{\text{bot}} - mg = m\frac{v^2}{r}$$

$$n_{\text{bot}} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

Substituting the values given for the speed and radius gives

$$n_{\text{bot}} = mg\left(1 + \frac{(225\text{ m/s})^2}{(2.70 \times 10^3\text{ m})(9.80\text{ m/s}^2)}\right) = \boxed{2.91mg}$$

Hence, the magnitude of the force $\mathbf{n}_{\text{bot}}$ exerted by the seat on the pilot is *greater* than the weight of the pilot by a factor of 2.91. This means that the pilot experiences an apparent weight that is greater than his true weight by a factor of 2.91.

(B) The free-body diagram for the pilot at the top of the loop is shown in Figure 6.7c. As we noted earlier, both the gravitational force exerted by the Earth and the force $\mathbf{n}_{\text{top}}$ exerted by the seat on the pilot act downward, and so the net downward force that provides the centripetal acceleration has a magnitude $n_{\text{top}} + mg$. Applying Newton's second law yields

$$\sum F = n_{\text{top}} + mg = m\frac{v^2}{r}$$

$$n_{\text{top}} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

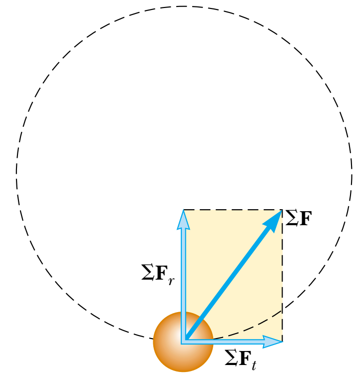
$$n_{\text{top}} = mg\left(\frac{(225\text{ m/s})^2}{(2.70 \times 10^3\text{ m})(9.80\text{ m/s}^2)} - 1\right)$$

$$= \boxed{0.913mg}$$

In this case, the magnitude of the force exerted by the seat on the pilot is *less* than his true weight by a factor of 0.913, and the pilot feels lighter. To finalize the problem, note that this is consistent with our prediction at the beginning of the solution.

6.2 Nonuniform Circular Motion

In Chapter 4 we found that if a particle moves with varying speed in a circular path, there is, in addition to the radial component of acceleration, a tangential component having magnitude dv/dt. Therefore, the force acting on the particle must also have a tangential and a radial component. Because the total acceleration is $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the total force exerted on the particle is $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$, as shown in Figure 6.8. The vector $\Sigma\mathbf{F}_r$ is directed toward the center of the circle and is responsible for the centripetal acceleration. The vector $\Sigma\mathbf{F}_t$ tangent to the circle is responsible for the tangential acceleration, which represents a change in the speed of the particle with time.



Active Figure 6.8 When the force acting on a particle moving in a circular path has a tangential component ΣF_t, the particle's speed changes. The total force exerted on the particle in this case is the vector sum of the radial force and the tangential force. That is, $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$.

At the Active Figures link at http://www.pse6.com, you can adjust the initial position of the particle and compare the component forces acting on the particle to those for a child swinging on a swing set.

Quick Quiz 6.3 Which of the following is *impossible* for a car moving in a circular path? (a) the car has tangential acceleration but no centripetal acceleration. (b) the car has centripetal acceleration but no tangential acceleration. (c) the car has both centripetal acceleration and tangential acceleration.

Quick Quiz 6.4 A bead slides freely along a *horizontal*, curved wire at constant speed, as shown in Figure 6.9. Draw the vectors representing the force exerted by the wire on the bead at points Ⓐ, Ⓑ, and Ⓒ.

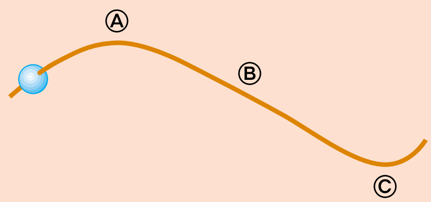


Figure 6.9 (Quick Quiz 6.4 and 6.5) A bead slides along a curved wire.

Quick Quiz 6.5 In Figure 6.9, the bead speeds up with constant tangential acceleration as it moves toward the right. Draw the vectors representing the force on the bead at points Ⓐ, Ⓑ, and Ⓒ.

Robin Smith / Getty Images

Passengers on a "corkscrew" roller coaster experience a radial force toward the center of the circular track and a tangential force due to gravity.

Example 6.7 Keep Your Eye on the Ball

Interactive

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O, as illustrated in Figure 6.10a. Determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Solution Unlike the situation in Example 6.6, the speed is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere. From the free-body diagram in Figure 6.10a, we see that the only forces acting on the sphere are the gravitational force $\mathbf{F}_g = m\mathbf{g}$ exerted by the Earth and the force $\mathbf{T}$ exerted by the cord. Now we resolve $\mathbf{F}_g$ into a tangential component $mg \sin\theta$ and a radial component $mg\cos\theta$. Applying Newton's second law to the forces acting on the sphere in the tangential direction yields

$$\sum F_t = mg\sin\theta = ma_t$$

$$a_t = g\sin\theta$$

This tangential component of the acceleration causes v to change in time because $a_t = dv/dt$.

Applying Newton's second law to the forces acting on the sphere in the radial direction and noting that both $\mathbf{T}$ and $\mathbf{a}_r$ are directed toward O, we obtain

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = \boxed{m\left(\frac{v^2}{R} + g\cos\theta\right)}$$

What If? What if we set the ball in motion with a slower speed? (A) What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Answer At the top of the path (Fig. 6.10b), where $\theta = 180°$, we have $\cos 180° = -1$, and the tension equation becomes

$$T_{top} = m\left(\frac{v_{top}^2}{R} - g\right)$$

Let us set $T_{top} = 0$. Then,

$$0 = m\left(\frac{v_{top}^2}{R} - g\right)$$

$$v_{top} = \sqrt{gR}$$

(B) What if we set the ball in motion such that the speed at the top is less than this value? What happens?

Answer In this case, the ball never reaches the top of the circle. At some point on the way up, the tension in the string goes to zero and the ball becomes a projectile. It follows a segment of a parabolic path over the top of its motion, rejoining the circular path on the other side when the tension becomes nonzero again.

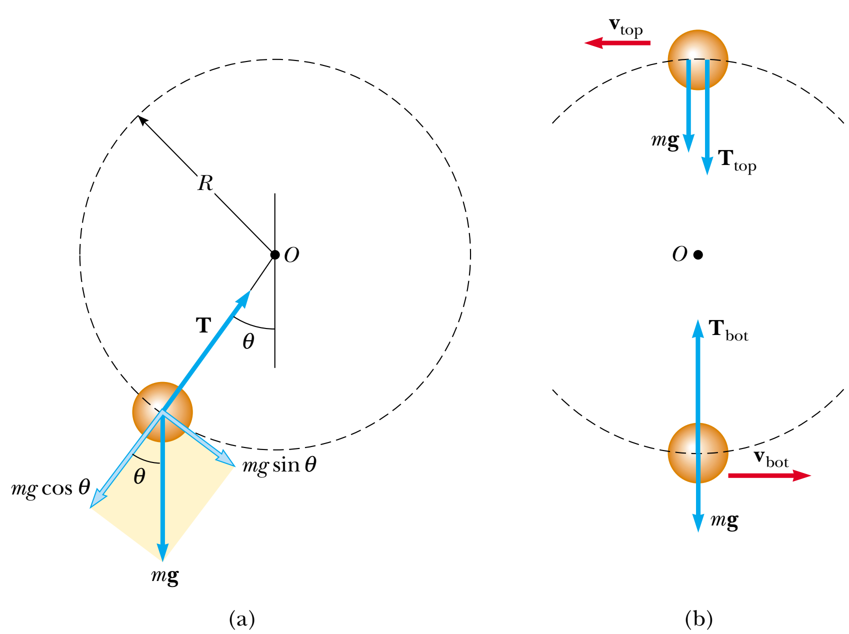


Figure 6.10 (a) Forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. (b) Forces acting on the sphere at the top and bottom of the circle. The tension is a maximum at the bottom and a minimum at the top.

Investigate these alternatives at the Interactive Worked Example link at **http://www.pse6.com.**

6.3 Motion in Accelerated Frames

When Newton's laws of motion were introduced in Chapter 5, we emphasized that they are valid only when observations are made in an inertial frame of reference. In this section, we analyze how Newton's second law is applied by an observer in a noninertial frame of reference, that is, one that is accelerating. For example, recall the discussion of the air hockey table on a train in Section 5.2. The train moving at constant velocity represents an inertial frame. The puck at rest remains at rest, and Newton's first law is obeyed. The accelerating train is not an inertial frame. According to you as the observer on the train, there appears to be no visible force on the puck, yet it accelerates from rest toward the back of the train, violating Newton's first law.

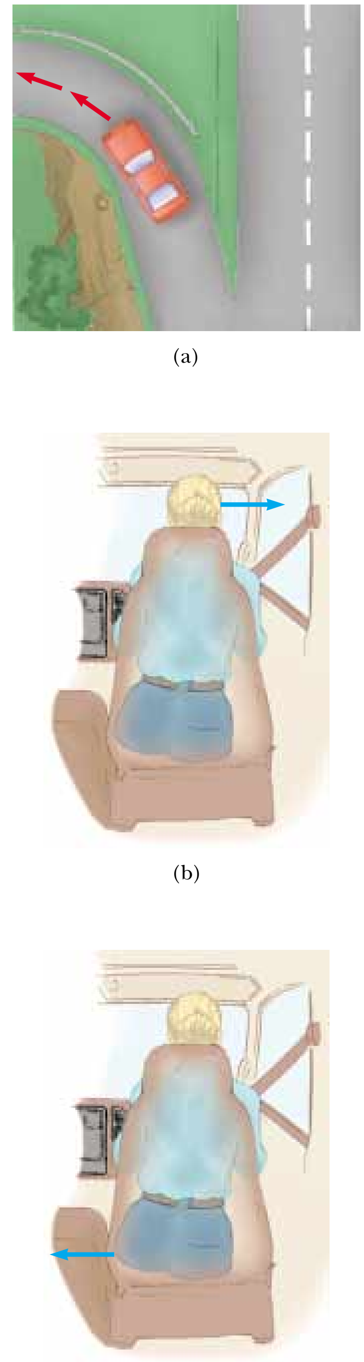
As an observer on the accelerating train, if you apply Newton's second law to the puck as it accelerates toward the back of the train, you might conclude that a force has acted on the puck to cause it to accelerate. We call an apparent force such as this a **fictitious force,** because it is due to an accelerated reference frame. Remember that real forces are always due to interactions between two objects. A fictitious force appears to act on an object in the same way as a real force, but you cannot identify a second object for a fictitious force.

The train example above describes a fictitious force due to a change in the speed of the train. Another fictitious force is due to the change in the *direction* of the velocity vector. To understand the motion of a system that is noninertial because of a change in direction, consider a car traveling along a highway at a high speed and approaching a curved exit ramp, as shown in Figure 6.11a. As the car takes the sharp left turn onto the ramp, a person sitting in the passenger seat slides to the right and hits the door. At that point, the force exerted by the door on the passenger keeps her from being ejected from the car. What causes her to move toward the door? A popular but incorrect explanation is that a force acting toward the right in Figure 6.11b pushes her outward. This is often called the "centrifugal force," but it is a fictitious force due to the acceleration associated with the changing direction of the car's velocity vector. (The driver also experiences this effect but wisely holds on to the steering wheel to keep from sliding to the right.)

The phenomenon is correctly explained as follows. Before the car enters the ramp, the passenger is moving in a straight-line path. As the car enters the ramp and travels a curved path, the passenger tends to move along the original straight-line path. This is in accordance with Newton's first law: the natural tendency of an object is to continue moving in a straight line. However, if a sufficiently large force (toward the center of curvature) acts on the passenger, as in Figure 6.11c, she moves in a curved path along with the car. This force is the force of friction between her and the car seat. If this friction force is not large enough, she slides to the right as the seat turns to the left under her. Eventually, she encounters the door, which provides a force large enough to enable her to follow the same curved path as the car. She slides toward the door not because of an outward force but because **the force of friction is not sufficiently great to allow her to travel along the circular path followed by the car.**

Another interesting fictitious force is the "Coriolis force." This is an apparent force caused by changing the radial position of an object in a rotating coordinate system. For example, suppose you and a friend are on opposite sides of a rotating circular platform and you decide to throw a baseball to your friend. As Figure 6.12a shows, at $t = 0$ you throw the ball toward your friend, but by the time t_f when the ball has crossed the platform, your friend has moved to a new position.

Figure 6.12a represents what an observer would see if the ball is viewed while the observer is hovering at rest above the rotating platform. According to this observer, who is in an inertial frame, the ball follows a straight line, as it must according to Newton's first law. Now, however, consider the situation from your friend's viewpoint. Your friend is in a noninertial reference frame because he is undergoing a centripetal acceleration relative to the inertial frame of the Earth's surface. He starts off seeing the baseball coming toward him, but as it crosses the platform, it veers to one side, as shown in Figure 6.12b. Thus, your friend on the rotating platform claims that the ball



(a) (b) (c)

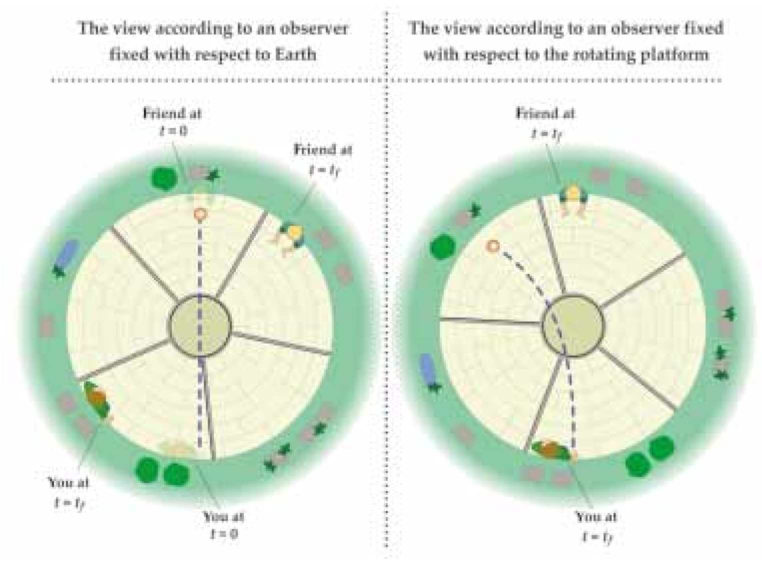
Figure 6.11 (a) A car approaching a curved exit ramp. What causes a front-seat passenger to move toward the right-hand door? (b) From the frame of reference of the passenger, a force appears to push her toward the right door, but this is a fictitious force. (c) Relative to the reference frame of the Earth, the car seat applies a leftward force to the passenger, causing her to change direction along with the rest of the car.

PITFALL PREVENTION

6.2 Centrifugal Force

The commonly heard phrase "centrifugal force" is described as a force pulling *outward* on an object moving in a circular path. If you are feeling a "centrifugal force" on a rotating carnival ride, what is the other object with which you are interacting? You cannot identify another object because this is a fictitious force that occurs as a result of your being in a noninertial reference frame.

At the Active Figures link at **http://www.pse6.com,** *you can observe the ball's path simultaneously from the reference frame of an inertial observer and from the reference frame of the rotating turntable.*



(a) (b)

Active Figure 6.12 (a) You and your friend sit at the edge of a rotating turntable. In this overhead view observed by someone in an inertial reference frame attached to the Earth, you throw the ball at $t = 0$ in the direction of your friend. By the time t_f that the ball arrives at the other side of the turntable, your friend is no longer there to catch it. According to this observer, the ball followed a straight line path, consistent with Newton's laws. (b) From the point of view of your friend, the ball veers to one side during its flight. Your friend introduces a fictitious force to cause this deviation from the expected path. This fictitious force is called the "Coriolis force."

does not obey Newton's first law and claims that a force is causing the ball to follow a curved path. This fictitious force is called the Coriolis force.

Fictitious forces may not be real forces, but they can have real effects. An object on your dashboard *really* slides off if you press the accelerator of your car. As you ride on a merry-go-round, you feel pushed toward the outside as if due to the fictitious "centrifugal force." You are likely to fall over and injure yourself if you walk along a radial line while the merry-go-round rotates. The Coriolis force due to the rotation of the Earth is responsible for rotations of hurricanes and for large-scale ocean currents.

Quick Quiz 6.6 Consider the passenger in the car making a left turn in Figure 6.11. Which of the following is correct about forces in the horizontal direction if the person is making contact with the right-hand door? (a) The passenger is in equilibrium between real forces acting to the right and real forces acting to the left. (b) The passenger is subject only to real forces acting to the right. (c) The passenger is subject only to real forces acting to the left. (d) None of these is true.

Example 6.8 Fictitious Forces in Linear Motion

A small sphere of mass m is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in Figure 6.13. The noninertial observer in Figure 6.13b claims that a force, which we know to be fictitious, must act in order to cause the observed deviation of the cord from the vertical. How is the magnitude of this force related to the acceleration of the boxcar measured by the inertial observer in Figure 6.13a?

Solution According to the inertial observer at rest (Fig. 6.13a), the forces on the sphere are the force $\mathbf{T}$ exerted by the cord and the gravitational force. The inertial observer concludes that the acceleration of the sphere is the same as that of the boxcar and that this acceleration is provided by the horizontal component of $\mathbf{T}$. Also, the vertical component of $\mathbf{T}$ balances the gravitational force because the sphere is in equilibrium in the vertical direction. Therefore,

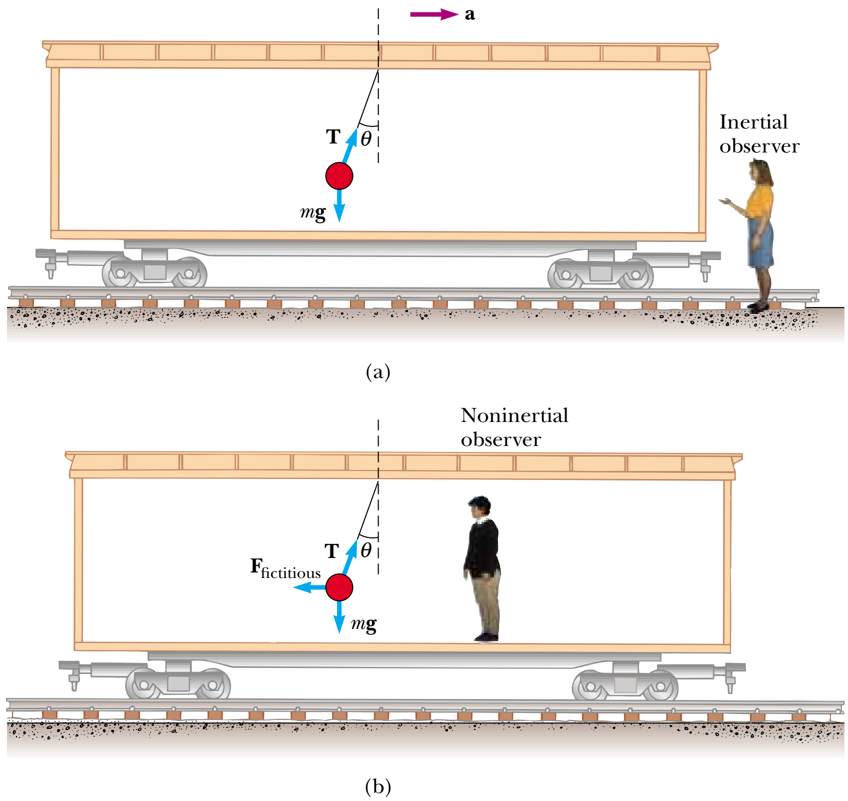


Figure 6.13 (Example 6.8) A small sphere suspended from the ceiling of a boxcar accelerating to the right is deflected as shown. (a) An inertial observer at rest outside the car claims that the acceleration of the sphere is provided by the horizontal component of **T**. (b) A noninertial observer riding in the car says that the net force on the sphere is zero and that the deflection of the cord from the vertical is due to a fictitious force $\mathbf{F}_{\text{fictitious}}$ that balances the horizontal component of **T**.

she writes Newton's second law as $\Sigma\mathbf{F} = \mathbf{T} + m\mathbf{g} = m\mathbf{a}$, which in component form becomes

$$\text{Inertial observer}\begin{cases}(1) \quad \sum F_x = T\sin\theta = ma \\ (2) \quad \sum F_y = T\cos\theta - mg = 0\end{cases}$$

According to the noninertial observer riding in the car (Fig. 6.13b), the cord also makes an angle θ with the vertical; however, to him the sphere is at rest and so its acceleration is zero. Therefore, he introduces a fictitious force in the horizontal direction to balance the horizontal component of **T** and claims that the net force on the sphere is *zero!* In this noninertial frame of reference, Newton's second law in component form yields

$$\text{Noninertial observer}\begin{cases}\sum F'_x = T\sin\theta - F_{\text{fictitious}} = 0 \\ \sum F'_y = T\cos\theta - mg = 0\end{cases}$$

We see that these expressions are equivalent to (1) and (2) if $F_{\text{fictitious}} = ma$, where a is the acceleration according to the inertial observer. If we were to make this substitution in the equation for F'_x above, the noninertial observer obtains the same mathematical results as the inertial observer. However, the physical interpretation of the deflection of the cord differs in the two frames of reference.

What If? Suppose the inertial observer wants to measure the acceleration of the train by means of the pendulum (the sphere hanging from the cord). How could she do this?

Answer Our intuition tells us that the angle θ that the cord makes with the vertical should increase as the acceleration increases. By solving (1) and (2) simultaneously for a, the inertial observer can determine the magnitude of the car's acceleration by measuring the angle θ and using the relationship $a = g\tan\theta$. Because the deflection of the cord from the vertical serves as a measure of acceleration, *a simple pendulum can be used as an accelerometer.*

Example 6.9 Fictitious Force in a Rotating System

Suppose a block of mass m lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown in Figure 6.14. How would each of the observers write Newton's second law for the block?

Solution According to an inertial observer (Fig. 6.14a), if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r, where v is its linear speed. The inertial observer concludes that this centripetal acceleration is provided by the force **T** exerted by the string and writes Newton's second law as $T = mv^2/r$.

According to a noninertial observer attached to the turntable (Fig 6.14b), the block is at rest and its acceleration is zero. Therefore, she must introduce a fictitious outward force of magnitude mv^2/r to balance the inward force exerted by the string. According to her, the net force on the block is zero, and she writes Newton's second law as $T - mv^2/r = 0$.

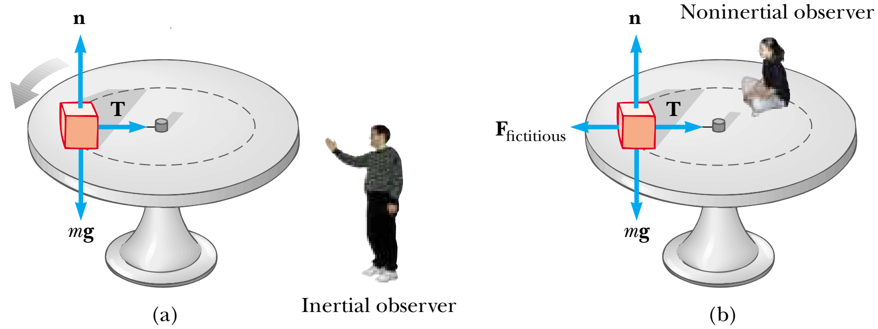


Figure 6.14 (Example 6.9) A block of mass m connected to a string tied to the center of a rotating turntable. (a) The inertial observer claims that the force causing the circular motion is provided by the force **T** exerted by the string on the block. (b) The noninertial observer claims that the block is not accelerating, and therefore she introduces a fictitious force of magnitude mv^2/r that acts outward and balances the force **T**.

6.4 Motion in the Presence of Resistive Forces

In the preceding chapter we described the force of kinetic friction exerted on an object moving on some surface. We completely ignored any interaction between the object and the medium through which it moves. Now let us consider the effect of that medium, which can be either a liquid or a gas. The medium exerts a **resistive force R** on the object moving through it. Some examples are the air resistance associated with moving vehicles (sometimes called *air drag*) and the viscous forces that act on objects moving through a liquid. The magnitude of **R** depends on factors such as the speed of the object, and the direction of **R** is always opposite the direction of motion of the object relative to the medium. Furthermore, the magnitude of **R** nearly always increases with increasing speed.

The magnitude of the resistive force can depend on speed in a complex way, and here we consider only two situations. In the first situation, we assume the resistive force is proportional to the speed of the moving object; this assumption is valid for objects falling slowly through a liquid and for very small objects, such as dust particles, moving through air. In the second situation, we assume a resistive force that is proportional to the square of the speed of the moving object; large objects, such as a skydiver moving through air in free fall, experience such a force.

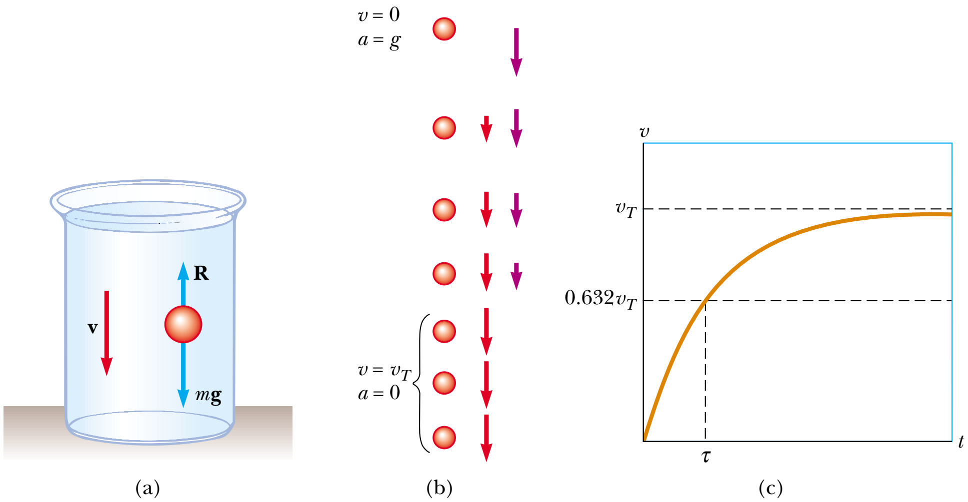
Resistive Force Proportional to Object Speed

If we assume that the resistive force acting on an object moving through a liquid or gas is proportional to the object's speed, then the resistive force can be expressed as

$$\mathbf{R} = -b\mathbf{v} \tag{6.2}$$

where **v** is the velocity of the object and b is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object. If the object is a sphere of radius r, then b is proportional to r. The negative sign indicates that **R** is in the opposite direction to **v**.

Consider a small sphere of mass m released from rest in a liquid, as in Figure 6.15a. Assuming that the only forces acting on the sphere are the resistive force $\mathbf{R} = -b\mathbf{v}$ and



Active Figure 6.15 (a) A small sphere falling through a liquid. (b) Motion diagram of the sphere as it falls. (c) Speed–time graph for the sphere. The sphere reaches a maximum (or terminal) speed v_T, and the time constant τ is the time interval during which it reaches a speed of $0.632v_T$.

At the Active Figures link at http://www.pse6.com, you can vary the size and mass of the sphere and the viscosity (resistance to flow) of the surrounding medium, then observe the effects on the sphere's motion and its speed–time graph.

the gravitational force $\mathbf{F}_g$, let us describe its motion.[1] Applying Newton's second law to the vertical motion, choosing the downward direction to be positive, and noting that $\Sigma F_y = mg - bv$, we obtain

$$mg - bv = ma = m\frac{dv}{dt} \tag{6.3}$$

where the acceleration dv/dt is downward. Solving this expression for the acceleration gives

$$\frac{dv}{dt} = g - \frac{b}{m}v \tag{6.4}$$

This equation is called a *differential equation,* and the methods of solving it may not be familiar to you as yet. However, note that initially when $v = 0$, the magnitude of the resistive force bv is also zero, and the acceleration dv/dt is simply g. As t increases, the magnitude of the resistive force increases and the acceleration decreases. The acceleration approaches zero when the magnitude of the resistive force approaches the sphere's weight. In this situation, the speed of the sphere approaches its **terminal speed** v_T. In reality, the sphere only *approaches* terminal speed but never *reaches* terminal speed.

Terminal speed

We can obtain the terminal speed from Equation 6.3 by setting $a = dv/dt = 0$. This gives

$$mg - bv_T = 0 \qquad \text{or} \qquad v_T = \frac{mg}{b}$$

The expression for v that satisfies Equation 6.4 with $v = 0$ at $t = 0$ is

$$v = \frac{mg}{b}(1 - e^{-bt/m}) = v_T(1 - e^{-t/\tau}) \tag{6.5}$$

This function is plotted in Figure 6.15c. The symbol e represents the base of the natural logarithm, and is also called *Euler's number:* $e = 2.718\ 28$. The **time constant** $\tau = m/b$ (Greek letter tau) is the time at which the sphere released from rest reaches 63.2% of its terminal speed. This can be seen by noting that when $t = \tau$, Equation 6.5 yields $v = 0.632v_T$.

[1] There is also a *buoyant force* acting on the submerged object. This force is constant, and its magnitude is equal to the weight of the displaced liquid. This force changes the apparent weight of the sphere by a constant factor, so we will ignore the force here. We discuss buoyant forces in Chapter 14.

We can check that Equation 6.5 is a solution to Equation 6.4 by direct differentiation:

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{mg}{b} - \frac{mg}{b}e^{-bt/m}\right) = -\frac{mg}{b}\frac{d}{dt}e^{-bt/m} = ge^{-bt/m}$$

(See Appendix Table B.4 for the derivative of e raised to some power.) Substituting into Equation 6.4 both this expression for dv/dt and the expression for v given by Equation 6.5 shows that our solution satisfies the differential equation.

Example 6.10 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time at which the sphere reaches 90.0% of its terminal speed.

Solution Because the terminal speed is given by $v_T = mg/b$, the coefficient b is

$$b = \frac{mg}{v_T} = \frac{(2.00\text{ g})(980\text{ cm/s}^2)}{5.00\text{ cm/s}} = 392\text{ g/s}$$

Therefore, the time constant τ is

$$\tau = \frac{m}{b} = \frac{2.00\text{ g}}{392\text{ g/s}} = \boxed{5.10 \times 10^{-3}\text{ s}}$$

The speed of the sphere as a function of time is given by Equation 6.5. To find the time t at which the sphere reaches a speed of $0.900v_T$, we set $v = 0.900v_T$ in Equation 6.5 and solve for t:

$$0.900v_T = v_T(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3}\text{ s})$$

$$= 11.7 \times 10^{-3}\text{ s} = \boxed{11.7\text{ ms}}$$

Thus, the sphere reaches 90.0% of its terminal speed in a very short time interval.

Air Drag at High Speeds

For objects moving at high speeds through air, such as airplanes, sky divers, cars, and baseballs, the resistive force is approximately proportional to the square of the speed. In these situations, the magnitude of the resistive force can be expressed as

$$R = \tfrac{1}{2}D\rho Av^2 \tag{6.6}$$

where ρ is the density of air, A is the cross-sectional area of the moving object measured in a plane perpendicular to its velocity, and D is a dimensionless empirical quantity called the *drag coefficient*. The drag coefficient has a value of about 0.5 for spherical objects but can have a value as great as 2 for irregularly shaped objects.

Let us analyze the motion of an object in free-fall subject to an upward air resistive force of magnitude $R = \frac{1}{2}D\rho Av^2$. Suppose an object of mass m is released from rest. As Figure 6.16 shows, the object experiences two external forces:[2] the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the upward resistive force $\mathbf{R}$. Hence, the magnitude of the net force is

$$\sum F = mg - \tfrac{1}{2}D\rho Av^2 \tag{6.7}$$

where we have taken downward to be the positive vertical direction. Combining $\Sigma F = ma$ with Equation 6.7, we find that the object has a downward acceleration of magnitude

$$a = g - \left(\frac{D\rho A}{2m}\right)v^2 \tag{6.8}$$

We can calculate the terminal speed v_T by using the fact that when the gravitational force is balanced by the resistive force, the net force on the object is zero and therefore its acceleration is zero. Setting $a = 0$ in Equation 6.8 gives

$$g - \left(\frac{D\rho A}{2m}\right)v_T^{\,2} = 0$$

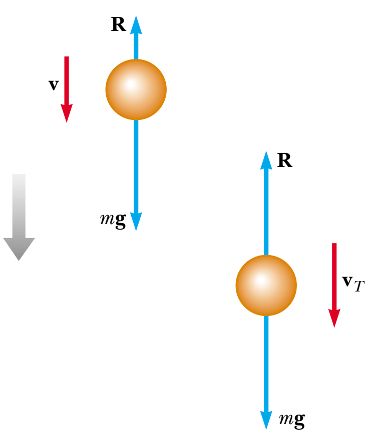


Figure 6.16 An object falling through air experiences a resistive force $\mathbf{R}$ and a gravitational force $\mathbf{F}_g = m\mathbf{g}$. The object reaches terminal speed (on the right) when the net force acting on it is zero, that is, when $\mathbf{R} = -\mathbf{F}_g$ or $R = mg$. Before this occurs, the acceleration varies with speed according to Equation 6.8.

[2] There is also an upward buoyant force that we neglect.

Table 6.1

Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m^2)	v_T (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0

so that,

$$v_T = \sqrt{\frac{2mg}{D\rho A}} \tag{6.9}$$

Using this expression, we can determine how the terminal speed depends on the dimensions of the object. Suppose the object is a sphere of radius r. In this case, $A \propto r^2$ (from $A = \pi r^2$) and $m \propto r^3$ (because the mass is proportional to the volume of the sphere, which is $V = \frac{4}{3}\pi r^3$). Therefore, $v_T \propto \sqrt{r}$.

Table 6.1 lists the terminal speeds for several objects falling through air.

Quick Quiz 6.7 A baseball and a basketball, having the same mass, are dropped through air from rest such that their bottoms are initially at the same height above the ground, on the order of 1 m or more. Which one strikes the ground first? (a) the baseball (b) the basketball (c) both strike the ground at the same time.

Conceptual Example 6.11 The Sky Surfer

Consider a sky surfer (Fig. 6.17) who jumps from a plane with her feet attached firmly to her surfboard, does some tricks, and then opens her parachute. Describe the forces acting on her during these maneuvers.

Solution When the surfer first steps out of the plane, she has no vertical velocity. The downward gravitational force causes her to accelerate toward the ground. As her downward speed increases, so does the upward resistive force exerted by the air on her body and the board. This upward force reduces their acceleration, and so their speed increases more slowly. Eventually, they are going so fast that the upward resistive force matches the downward gravitational force. Now the net force is zero and they no longer accelerate, but reach their terminal speed. At some point after reaching terminal speed, she opens her parachute, resulting in a drastic increase in the upward resistive force. The net force (and thus the acceleration) is now upward, in the direction opposite the direction of the velocity. This causes the downward velocity to decrease rapidly; this means the resistive force on the chute also decreases. Eventually the upward resistive force and the downward gravitational force balance each other and a much smaller terminal speed is reached, permitting a safe landing.

(Contrary to popular belief, the velocity vector of a sky diver never points upward. You may have seen a videotape in which a sky diver appears to "rocket" upward once the chute opens. In fact, what happens is that the diver slows down while the person holding the camera continues falling at high speed.)

Figure 6.17 (Conceptual Example 6.11) A sky surfer.

Example 6.12 Falling Coffee Filters

The dependence of resistive force on speed is an empirical relationship. In other words, it is based on observation rather than on a theoretical model. Imagine an experiment in which we drop a series of stacked coffee filters, and measure their terminal speeds. Table 6.2 presents data for these coffee filters as they fall through the air. The time constant τ is small, so that a dropped filter quickly reaches terminal speed. Each filter has a mass of 1.64 g. When the filters are nested together, they stack in such a way that the front-facing surface area does not increase. Determine the relationship between the resistive force exerted by the air and the speed of the falling filters.

Pleated coffee filters can be nested together so that the force of air resistance can be studied.

Solution At terminal speed, the upward resistive force balances the downward gravitational force. So, a single filter falling at its terminal speed experiences a resistive force of

$$R = mg = (1.64\ \text{g})\left(\frac{1\ \text{kg}}{100\,0\ \text{g}}\right)(9.80\ \text{m/s}^2) = 0.016\,1\ \text{N}$$

Two filters nested together experience 0.032 2 N of resistive force, and so forth. A graph of the resistive force on the filters as a function of terminal speed is shown in Figure 6.18a. A straight line would not be a good fit, indicating that the resistive force is *not* proportional to the speed. The behavior is more clearly seen in Figure 6.18b, in which the resistive force is plotted as a function of the square of the terminal speed. This indicates a proportionality of the resistive force to the *square* of the speed, as suggested by Equation 6.6.

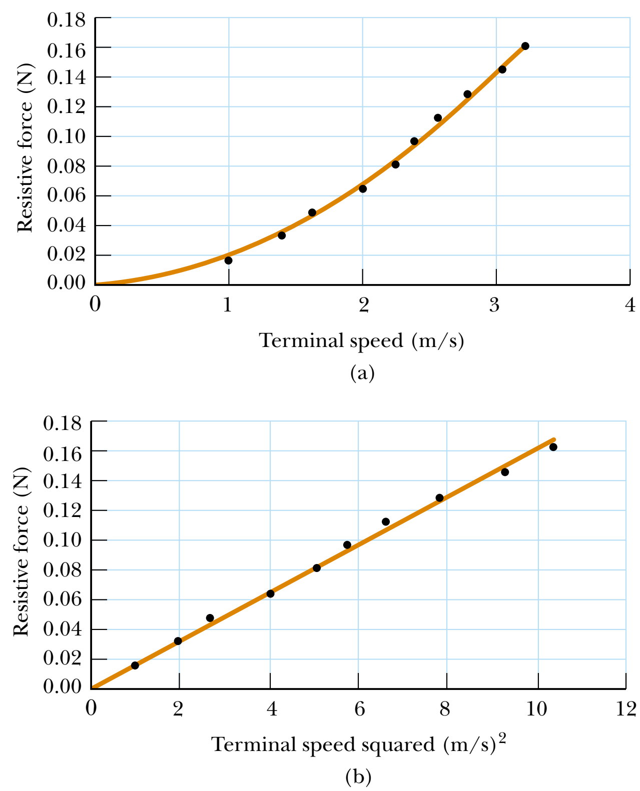


Figure 6.18 (Example 6.12) (a) Relationship between the resistive force acting on falling coffee filters and their terminal speed. The curved line is a second-order polynomial fit. (b) Graph relating the resistive force to the square of the terminal speed. The fit of the straight line to the data points indicates that the resistive force is proportional to the terminal speed squared. Can you find the proportionality constant?

Table 6.2

Terminal Speed for Stacked Coffee Filters

Number of Filters	v_T (m/s)[a]
1	1.01
2	1.40
3	1.63
4	2.00
5	2.25
6	2.40
7	2.57
8	2.80
9	3.05
10	3.22

[a] All values of v_T are approximate.

Example 6.13 Resistive Force Exerted on a Baseball

A pitcher hurls a 0.145-kg baseball past a batter at 40.2 m/s (= 90 mi/h). Find the resistive force acting on the ball at this speed.

Solution We do not expect the air to exert a huge force on the ball, and so the resistive force we calculate from Equation 6.6 should not be more than a few newtons.

First, we must determine the drag coefficient D. We do this by imagining that we drop the baseball and allow it to reach terminal speed. We solve Equation 6.9 for D and substitute the appropriate values for m, v_T, and A from Table 6.1. Taking the density of air as 1.20 kg/m^3, we obtain

$$D = \frac{2mg}{v_T{}^2\rho A} = \frac{2(0.145\text{ kg})(9.80\text{ m/s}^2)}{(43\text{ m/s})^2(1.20\text{ kg/m}^3)(4.2\times10^{-3}\text{ m}^2)}$$
$$= 0.305$$

This number has no dimensions. We have kept an extra digit beyond the two that are significant and will drop it at the end of our calculation.

We can now use this value for D in Equation 6.6 to find the magnitude of the resistive force:

$$R = \tfrac{1}{2}D\rho A v^2$$
$$= \tfrac{1}{2}(0.305)(1.20\text{ kg/m}^3)(4.2\times10^{-3}\text{ m}^2)(40.2\text{ m/s})^2$$
$$= \boxed{1.2\text{ N}}$$

6.5 Numerical Modeling in Particle Dynamics[3]

As we have seen in this and the preceding chapter, the study of the dynamics of a particle focuses on describing the position, velocity, and acceleration as functions of time. Cause-and-effect relationships exist among these quantities: Velocity causes position to change, and acceleration causes velocity to change. Because acceleration is the direct result of applied forces, any analysis of the dynamics of a particle usually begins with an evaluation of the net force acting on the particle.

Until now, we have used what is called the *analytical method* to investigate the position, velocity, and acceleration of a moving particle. This method involves the identification of well-behaved functional expressions for the position of a particle (such as the kinematic equations of Chapter 2), generated from algebraic manipulations or the techniques of calculus. Let us review this method briefly before learning about a second way of approaching problems in dynamics. (Because we confine our discussion to one-dimensional motion in this section, boldface notation will not be used for vector quantities.)

If a particle of mass m moves under the influence of a net force ΣF, Newton's second law tells us that the acceleration of the particle is $a = \Sigma F/m$. In general, we apply the analytical method to a dynamics problem using the following procedure:

1. Sum all the forces acting on the particle to find the net force ΣF.
2. Use this net force to determine the acceleration from the relationship $a = \Sigma F/m$.
3. Use this acceleration to determine the velocity from the relationship $dv/dt = a$.
4. Use this velocity to determine the position from the relationship $dx/dt = v$.

The following straightforward example illustrates this method.

Example 6.14 An Object Falling in a Vacuum—Analytical Method

Consider a particle falling in a vacuum under the influence of the gravitational force, as shown in Figure 6.19. Use the analytical method to find the acceleration, velocity, and position of the particle.

Solution The only force acting on the particle is the downward gravitational force of magnitude F_g, which is also the net force. Applying Newton's second law, we set the net force acting on the particle equal to the mass of the particle times its acceleration (taking upward to be the positive y direction):

$$F_g = ma_y = -mg$$

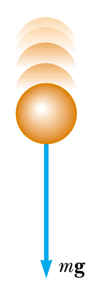


Figure 6.19 (Example 6.14) An object falling in vacuum under the influence of gravity.

[3] The authors are most grateful to Colonel James Head of the U.S. Air Force Academy for preparing this section.

Thus, $a_y = -g$, which means the acceleration is constant. Because $dv_y/dt = a_y$, we see that $dv_y/dt = -g$, which may be integrated to yield

$$v_y(t) = v_{yi} - gt$$

Then, because $v_y = dy/dt$, the position of the particle is obtained from another integration, which yields the well-known result

$$y(t) = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

In these expressions, y_i and v_{yi} represent the position and speed of the particle at $t_i = 0$.

The analytical method is straightforward for many physical situations. In the "real world," however, complications often arise that make analytical solutions difficult and perhaps beyond the mathematical abilities of most students taking introductory physics. For example, the net force acting on a particle may depend on the particle's position, as in cases where the gravitational acceleration varies with height. Or the force may vary with velocity, as in cases of resistive forces caused by motion through a liquid or gas.

Another complication arises because the expressions relating acceleration, velocity, position, and time are differential equations rather than algebraic ones. Differential equations are usually solved using integral calculus and other special techniques that introductory students may not have mastered.

When such situations arise, scientists often use a procedure called *numerical modeling* to study motion. The simplest numerical model is called the Euler method, after the Swiss mathematician Leonhard Euler (1707–1783).

The Euler Method

In the **Euler method** for solving differential equations, derivatives are approximated as ratios of finite differences. Considering a small increment of time Δt, we can approximate the relationship between a particle's speed and the magnitude of its acceleration as

$$a(t) \approx \frac{\Delta v}{\Delta t} = \frac{v(t + \Delta t) - v(t)}{\Delta t}$$

Then the speed $v(t + \Delta t)$ of the particle at the end of the time interval Δt is approximately equal to the speed $v(t)$ at the beginning of the time interval plus the magnitude of the acceleration during the interval multiplied by Δt:

$$v(t + \Delta t) \approx v(t) + a(t)\,\Delta t \tag{6.10}$$

Because the acceleration is a function of time, this estimate of $v(t + \Delta t)$ is accurate only if the time interval Δt is short enough such that the change in acceleration during the interval is very small (as is discussed later). Of course, Equation 6.10 is exact if the acceleration is constant.

The position $x(t + \Delta t)$ of the particle at the end of the interval Δt can be found in the same manner:

$$v(t) \approx \frac{\Delta x}{\Delta t} = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) \approx x(t) + v(t)\,\Delta t \tag{6.11}$$

You may be tempted to add the term $\frac{1}{2}a(\Delta t)^2$ to this result to make it look like the familiar kinematics equation, but this term is not included in the Euler method because Δt is assumed to be so small that $(\Delta t)^2$ is nearly zero.

If the acceleration at any instant t is known, the particle's velocity and position at a time $t + \Delta t$ can be calculated from Equations 6.10 and 6.11. The calculation then proceeds in a series of finite steps to determine the velocity and position at any later time.

Table 6.3

The Euler Method for Solving Dynamics Problems

Step	Time	Position	Velocity	Acceleration
0	t_0	x_0	v_0	$a_0 = F(x_0, v_0, t_0)/m$
1	$t_1 = t_0 + \Delta t$	$x_1 = x_0 + v_0 \Delta t$	$v_1 = v_0 + a_0 \Delta t$	$a_1 = F(x_1, v_1, t_1)/m$
2	$t_2 = t_1 + \Delta t$	$x_2 = x_1 + v_1 \Delta t$	$v_2 = v_1 + a_1 \Delta t$	$a_2 = F(x_2, v_2, t_2)/m$
3	$t_3 = t_2 + \Delta t$	$x_3 = x_2 + v_2 \Delta t$	$v_3 = v_2 + a_2 \Delta t$	$a_3 = F(x_3, v_3, t_3)/m$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	t_n	x_n	v_n	a_n

The acceleration is determined from the net force acting on the particle, and this force may depend on position, velocity, or time:

$$a(x, v, t) = \frac{\sum F(x, v, t)}{m} \tag{6.12}$$

It is convenient to set up the numerical solution to this kind of problem by numbering the steps and entering the calculations in a table. Table 6.3 illustrates how to do this in an orderly way. Many small increments can be taken, and accurate results can usually be obtained with the help of a computer. The equations provided in the table can be entered into a spreadsheet and the calculations performed row by row to determine the velocity, position, and acceleration as functions of time. The calculations can also be carried out using a programming language, or with commercially available mathematics packages for personal computers. Graphs of velocity versus time or position versus time can be displayed to help you visualize the motion.

One advantage of the Euler method is that the dynamics is not obscured—the fundamental relationships between acceleration and force, velocity and acceleration, and position and velocity are clearly evident. Indeed, these relationships form the heart of the calculations. There is no need to use advanced mathematics, and the basic physics governs the dynamics.

The Euler method is completely reliable for infinitesimally small time increments, but for practical reasons a finite increment size must be chosen. For the finite difference approximation of Equation 6.10 to be valid, the time increment must be small enough that the acceleration can be approximated as being constant during the increment. We can determine an appropriate size for the time increment by examining the particular problem being investigated. The criterion for the size of the time increment may need to be changed during the course of the motion. In practice, however, we usually choose a time increment appropriate to the initial conditions and use the same value throughout the calculations.

The size of the time increment influences the accuracy of the result, but unfortunately it is not easy to determine the accuracy of an Euler-method solution without a knowledge of the correct analytical solution. One method of determining the accuracy of the numerical solution is to repeat the calculations with a smaller time increment and compare results. If the two calculations agree to a certain number of significant figures, you can assume that the results are correct to that precision.

Example 6.15 Euler and the Sphere in Oil Revisited

Consider the sphere falling in oil in Example 6.10. Using the Euler method, find the position and the acceleration of the sphere at the instant that the speed reaches 90.0% of terminal speed.

Solution The net force on the sphere is

$$\Sigma F = -mg + bv$$

Thus, the acceleration values in the last column of Table 6.3 are

$$a = \frac{\Sigma F(x, v, t)}{m} = \frac{-mg + bv}{m} = -g + \frac{bv}{m}$$

Choosing a time increment of 0.1 ms, the first few lines of the spreadsheet modeled after Table 6.3 look like Table 6.4. We see that the speed is increasing while the magnitude of the acceleration is decreasing due to the resistive force. We also see that the sphere does not fall very far in the first millisecond.

Further down the spreadsheet, as shown in Table 6.5, we find the instant at which the sphere reaches the speed $0.900v_T$, which is $0.900 \times 5.00 \text{ cm/s} = 4.50 \text{ cm/s}$. This calculation shows that this occurs at $t = 11.6$ ms, which agrees within its uncertainty with the value obtained in Example 6.10. The 0.1-ms difference in the two values is due to the approximate nature of the Euler method. If a smaller time increment were used, the instant at which the speed reaches $0.900v_T$ approaches the value calculated in Example 6.10.

From Table 6.5, we see that the position and acceleration of the sphere when it reaches a speed of $0.900v_T$ are

$$y = \boxed{-0.035 \text{ cm}} \qquad \text{and} \qquad a = \boxed{-99 \text{ cm/s}^2}$$

Table 6.4

The Sphere Begins to Fall in Oil

Step	Time (ms)	Position (cm)	Velocity (cm/s)	Acceleration (cm/s²)
0	0.0	0.000 0	0.0	−980.0
1	0.1	0.000 0	−0.10	−960.8
2	0.2	0.000 0	−0.19	−942.0
3	0.3	0.000 0	−0.29	−923.5
4	0.4	−0.000 1	−0.38	−905.4
5	0.5	−0.000 1	−0.47	−887.7
6	0.6	−0.000 1	−0.56	−870.3
7	0.7	−0.000 2	−0.65	−853.2
8	0.8	−0.000 3	−0.73	−836.5
9	0.9	−0.000 3	−0.82	−820.1
10	1.0	−0.000 4	−0.90	−804.0

Table 6.5

The Sphere Reaches 0.900 v_T

Step	Time (ms)	Position (cm)	Velocity (cm/s)	Acceleration (cm/s²)
110	11.0	−0.032 4	−4.43	−111.1
111	11.1	−0.032 8	−4.44	−108.9
112	11.2	−0.033 3	−4.46	−106.8
113	11.3	−0.033 7	−4.47	−104.7
114	11.4	−0.034 2	−4.48	−102.6
115	11.5	−0.034 6	−4.49	−100.6
116	11.6	−0.035 1	−4.50	−98.6
117	11.7	−0.035 5	−4.51	−96.7
118	11.8	−0.036 0	−4.52	−94.8
119	11.9	−0.036 4	−4.53	−92.9
120	12.0	−0.036 9	−4.54	−91.1

SUMMARY

Take a practice test for this chapter by clicking the Practice Test link at http://www.pse6.com.

Newton's second law applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is

$$\sum F = ma_c = \frac{mv^2}{r} \tag{6.1}$$

A particle moving in nonuniform circular motion has both a radial component of acceleration and a nonzero tangential component of acceleration. In the case of a particle rotating in a vertical circle, the gravitational force provides the tangential component of acceleration and part or all of the radial component of acceleration.

An observer in a noninertial (accelerating) frame of reference must introduce **fictitious forces** when applying Newton's second law in that frame. If these fictitious forces are properly defined, the description of motion in the noninertial frame is equivalent to that made by an observer in an inertial frame. However, the observers in the two frames do not agree on the causes of the motion.

An object moving through a liquid or gas experiences a speed-dependent **resistive force.** This resistive force, which opposes the motion relative to the medium, generally increases with speed. The magnitude of the resistive force depends on the size and shape of the object and on the properties of the medium through which the object is moving. In the limiting case for a falling object, when the magnitude of the resistive force equals the object's weight, the object reaches its **terminal speed.** **Euler's method** provides a means for analyzing the motion of a particle under the action of a force that is not simple.

QUESTIONS

1. Why does mud fly off a rapidly turning automobile tire?
2. Imagine that you attach a heavy object to one end of a spring, hold onto the other end of the spring, and then whirl the object in a horizontal circle. Does the spring stretch? If so, why? Discuss this in terms of the force causing the motion to be circular.
3. Describe a situation in which the driver of a car can have a centripetal acceleration but no tangential acceleration.
4. Describe the path of a moving body in the event that its acceleration is constant in magnitude at all times and (a) perpendicular to the velocity; (b) parallel to the velocity.
5. An object executes circular motion with constant speed whenever a net force of constant magnitude acts perpendicular to the velocity. What happens to the speed if the force is not perpendicular to the velocity?
6. Explain why the Earth is not spherical in shape and bulges at the equator.
7. Because the Earth rotates about its axis, it is a noninertial frame of reference. Assume the Earth is a uniform sphere. Why would the apparent weight of an object be greater at the poles than at the equator?
8. What causes a rotary lawn sprinkler to turn?
9. If someone told you that astronauts are weightless in orbit because they are beyond the pull of gravity, would you accept the statement? Explain.
10. It has been suggested that rotating cylinders about 10 mi in length and 5 mi in diameter be placed in space and used as colonies. The purpose of the rotation is to simulate gravity for the inhabitants. Explain this concept for producing an effective imitation of gravity.
11. Consider a rotating space station, spinning with just the right speed such that the centripetal acceleration on the inner surface is g. Thus, astronauts standing on this inner surface would feel pressed to the surface as if they were pressed into the floor because of the Earth's gravitational force. Suppose an astronaut in this station holds a ball above her head and "drops" it to the floor. Will the ball fall just like it would on the Earth?
12. A pail of water can be whirled in a vertical path such that none is spilled. Why does the water stay in the pail, even when the pail is above your head?
13. How would you explain the force that pushes a rider toward the side of a car as the car rounds a corner?
14. Why does a pilot tend to black out when pulling out of a steep dive?
15. The observer in the accelerating elevator of Example 5.8 would claim that the "weight" of the fish is T, the scale reading. This is obviously wrong. Why does this observation differ from that of a person outside the elevator, at rest with respect to the Earth?
16. If you have ever taken a ride in an express elevator of a high-rise building, you may have experienced a nauseating sensation of heaviness or lightness depending on the direction of the acceleration. Explain these sensations. Are we truly weightless in free-fall?
17. A falling sky diver reaches terminal speed with her parachute closed. After the parachute is opened, what parameters change to decrease this terminal speed?
18. Consider a small raindrop and a large raindrop falling through the atmosphere. Compare their terminal speeds. What are their accelerations when they reach terminal speed?
19. On long journeys, jet aircraft usually fly at high altitudes of about 30 000 ft. What is the main advantage of flying at these altitudes from an economic viewpoint?
20. Analyze the motion of a rock falling through water in terms of its speed and acceleration as it falls. Assume that the resistive force acting on the rock increases as the speed increases.
21. "If the current position and velocity of every particle in the Universe were known, together with the laws describing the forces that particles exert on one another, then the whole future of the Universe could be calculated. The future is determinate and preordained. Free will is an illusion." Do you agree with this thesis? Argue for or against it.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 6.1 Newton's Second Law Applied to Uniform Circular Motion

1. A light string can support a stationary hanging load of 25.0 kg before breaking. A 3.00-kg object attached to the string rotates on a horizontal, frictionless table in a circle of radius 0.800 m, while the other end of the string is held fixed. What range of speeds can the object have before the string breaks?

2. A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the total force on the driver has magnitude 130 N. What is the total vector force on the driver if the speed is 18.0 m/s instead?

3. In the Bohr model of the hydrogen atom, the speed of the electron is approximately 2.20×10^6 m/s. Find (a) the force acting on the electron as it revolves in a circular orbit of radius 0.530×10^{-10} m and (b) the centripetal acceleration of the electron.

4. In a cyclotron (one type of particle accelerator), a deuteron (of atomic mass 2.00 u) reaches a final speed of 10.0% of the speed of light while moving in a circular path of radius 0.480 m. The deuteron is maintained in the circular path by a magnetic force. What magnitude of force is required?

5. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What force causes the centripetal acceleration when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?

6. Whenever two *Apollo* astronauts were on the surface of the Moon, a third astronaut orbited the Moon. Assume the orbit to be circular and 100 km above the surface of the Moon, where the acceleration due to gravity is 1.52 m/s^2. The radius of the Moon is 1.70×10^6 m. Determine (a) the astronaut's orbital speed, and (b) the period of the orbit.

7. A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding?

8. The cornering performance of an automobile is evaluated on a skidpad, where the maximum speed that a car can maintain around a circular path on a dry, flat surface is measured. Then the centripetal acceleration, also called the lateral acceleration, is calculated as a multiple of the free-fall acceleration g. The main factors affecting the performance are the tire characteristics and the suspension system of the car. A Dodge Viper GTS can negotiate a skidpad of radius 61.0 m at 86.5 km/h. Calculate its maximum lateral acceleration.

9. Consider a conical pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of 5.00° with the vertical (Fig. P6.9). Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum and (b) the radial acceleration of the bob.

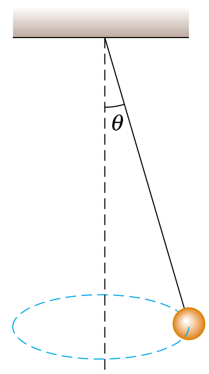


Figure P6.9

10. A car initially traveling eastward turns north by traveling in a circular path at uniform speed as in Figure P6.10. The length of the arc *ABC* is 235 m, and the car completes the turn in 36.0 s. (a) What is the acceleration when the car is at *B* located at an angle of 35.0°? Express your answer in terms of the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. Determine (b) the car's average speed and (c) its average acceleration during the 36.0-s interval.

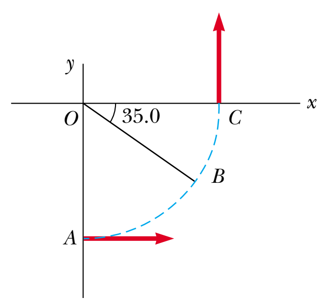


Figure P6.10

11. A 4.00-kg object is attached to a vertical rod by two strings, as in Figure P6.11. The object rotates in a horizontal circle at constant speed 6.00 m/s. Find the tension in (a) the upper string and (b) the lower string.

12. Casting of molten metal is important in many industrial processes. *Centrifugal casting* is used for manufacturing pipes, bearings and many other structures. A variety of sophisticated techniques have been invented, but the basic idea is as illustrated in Figure P6.12. A cylindrical enclosure is rotated rapidly and steadily about a horizontal axis. Molten metal is poured into the rotating cylinder and then cooled, forming the finished product. Turning the cylin-

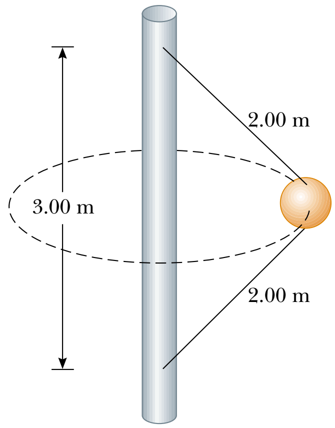


Figure P6.11

der at a high rotation rate forces the solidifying metal strongly to the outside. Any bubbles are displaced toward the axis, so unwanted voids will not be present in the casting. Sometimes it is desirable to form a composite casting, such as for a bearing. Here a strong steel outer surface is poured, followed by an inner lining of special low-friction metal. In some applications a very strong metal is given a coating of corrosion-resistant metal. Centrifugal casting results in strong bonding between the layers.

Suppose that a copper sleeve of inner radius 2.10 cm and outer radius 2.20 cm is to be cast. To eliminate bubbles and give high structural integrity, the centripetal acceleration of each bit of metal should be $100g$. What rate of rotation is required? State the answer in revolutions per minute.

Section 6.2 Nonuniform Circular Motion

13. A 40.0-kg child swings in a swing supported by two chains, each 3.00 m long. If the tension in each chain at the lowest point is 350 N, find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)

14. A child of mass m swings in a swing supported by two chains, each of length R. If the tension in each chain at the lowest point is T, find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)

15. Tarzan (m = 85.0 kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) will be 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1 000 N. Does he make it safely across the river?

16. A hawk flies in a horizontal arc of radius 12.0 m at a constant speed of 4.00 m/s. (a) Find its centripetal acceleration. (b) It continues to fly along the same horizontal arc but increases its speed at the rate of 1.20 m/s^2. Find the acceleration (magnitude and direction) under these conditions.

17. A pail of water is rotated in a vertical circle of radius 1.00 m. What is the minimum speed of the pail at the top of the circle if no water is to spill out?

18. A 0.400-kg object is swung in a vertical circular path on a string 0.500 m long. If its speed is 4.00 m/s at the top of the circle, what is the tension in the string there?

19. A roller coaster car (Fig. P6.19) has a mass of 500 kg when fully loaded with passengers. (a) If the vehicle has a speed of 20.0 m/s at point Ⓐ, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the vehicle can have at Ⓑ and still remain on the track?

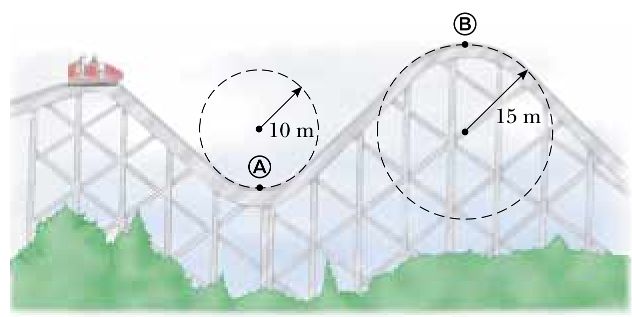


Figure P6.19

20. A roller coaster at the Six Flags Great America amusement park in Gurnee, IL, incorporates some clever design technology and some basic physics. Each vertical loop, instead of being circular, is shaped like a teardrop (Fig. P6.20). The cars ride on the inside of the loop at the top, and the speeds are high enough to ensure that the cars remain on the track. The biggest loop is 40.0 m high, with a maximum speed of 31.0 m/s (nearly 70 mi/h) at the bottom. Suppose

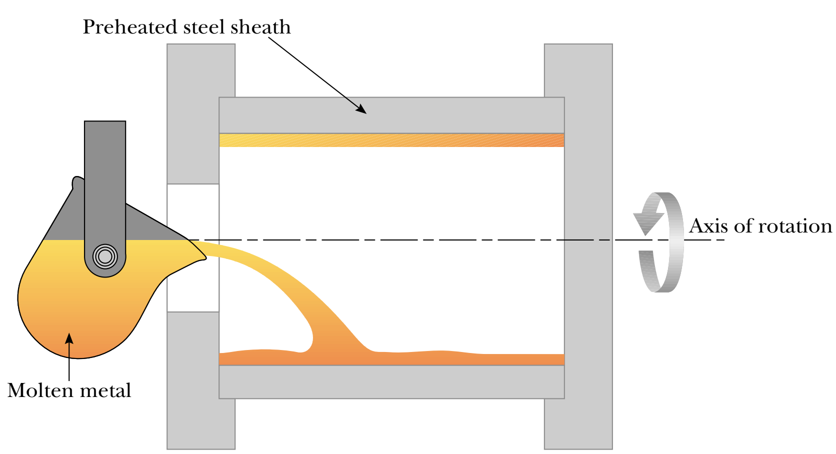


Figure P6.12

the speed at the top is 13.0 m/s and the corresponding centripetal acceleration is $2g$. (a) What is the radius of the arc of the teardrop at the top? (b) If the total mass of a car plus the riders is M, what force does the rail exert on the car at the top? (c) Suppose the roller coaster had a circular loop of radius 20.0 m. If the cars have the same speed, 13.0 m/s at the top, what is the centripetal acceleration at the top? Comment on the normal force at the top in this situation.

Frank Cezus / Getty Images

Figure P6.20

Section 6.3 Motion in Accelerated Frames

21. An object of mass 5.00 kg, attached to a spring scale, rests on a frictionless, horizontal surface as in Figure P6.21. The spring scale, attached to the front end of a boxcar, has a constant reading of 18.0 N when the car is in motion. (a) If the spring scale reads zero when the car is at rest, determine the acceleration of the car. (b) What constant reading will the spring scale show if the car moves with constant velocity? (c) Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.

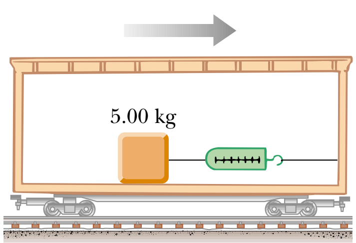


Figure P6.21

22. If the coefficient of static friction between your coffee cup and the horizontal dashboard of your car is $\mu_s = 0.800$, how fast can you drive on a horizontal roadway around a right turn of radius 30.0 m before the cup starts to slide? If you go too fast, in what direction will the cup slide relative to the dashboard?

23. A 0.500-kg object is suspended from the ceiling of an accelerating boxcar as in Figure 6.13. If $a = 3.00\ \text{m/s}^2$, find (a) the angle that the string makes with the vertical and (b) the tension in the string.

24. A small container of water is placed on a carousel inside a microwave oven, at a radius of 12.0 cm from the center. The turntable rotates steadily, turning through one revolution in each 7.25 s. What angle does the water surface make with the horizontal?

25. A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of 591 N. As the elevator later stops, the scale reading is 391 N. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.

26. The Earth rotates about its axis with a period of 24.0 h. Imagine that the rotational speed can be increased. If an object at the equator is to have zero apparent weight, (a) what must the new period be? (b) By what factor would the speed of the object be increased when the planet is rotating at the higher speed? Note that the apparent weight of the object becomes zero when the normal force exerted on it is zero.

27. A small block is at rest on the floor at the front of a railroad boxcar that has length ℓ. The coefficient of kinetic friction between the floor of the car and the block is μ_k. The car, originally at rest, begins to move with acceleration a. The block slides back horizontally until it hits the back wall of the car. At that moment, what is its speed (a) relative to the car? (b) relative to Earth?

28. A student stands in an elevator that is continuously accelerating upward with acceleration a. Her backpack is sitting on the floor next to the wall. The width of the elevator car is L. The student gives her backpack a quick kick at $t = 0$, imparting to it speed v, and making it slide across the elevator floor. At time t, the backpack hits the opposite wall. Find the coefficient of kinetic friction μ_k between the backpack and the elevator floor.

29. A child on vacation wakes up. She is lying on her back. The tension in the muscles on both sides of her neck is 55.0 N as she raises her head to look past her toes and out the motel window. Finally it is not raining! Ten minutes later she is screaming feet first down a water slide at terminal speed 5.70 m/s, riding high on the outside wall of a horizontal curve of radius 2.40 m (Figure P6.29). She raises her head to look forward past her toes; find the tension in the muscles on both sides of her neck.

Figure P6.29

30. One popular design of a household juice machine is a conical, perforated stainless steel basket 3.30 cm high with a closed bottom of diameter 8.00 cm and open top of diameter 13.70 cm that spins at 20 000 revolutions per minute about a vertical axis (Figure P6.30). Solid pieces of fruit are chopped into granules by cutters at the bottom of the spinning cone. Then the fruit granules rapidly make their way to the sloping surface where the juice is extracted to the outside of the cone through the mesh perforations. The dry pulp spirals upward along the slope to be ejected from the top of the cone. The juice is collected in an enclosure immediately surrounding the sloped surface of the cone. (a) What centripetal acceleration does a bit of fruit experience when it is spinning with the basket at a point midway between the top and bottom? Express the answer as a multiple of g. (b) Observe that the weight of the fruit is a negligible force. What is the normal force on 2.00 g of fruit at that point? (c) If the effective coefficient of kinetic friction between the fruit and the cone is 0.600, with what acceleration relative to the cone will the bit of fruit start to slide up the wall of the cone at that point, after being temporarily stuck?

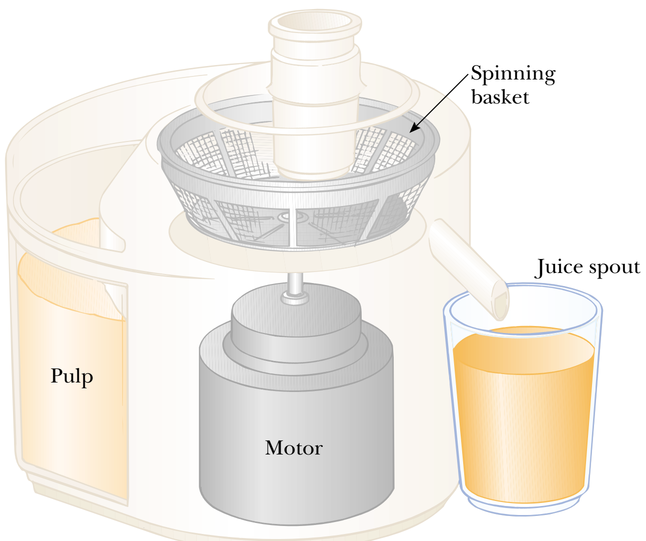


Figure P6.30

31. A plumb bob does not hang exactly along a line directed to the center of the Earth's rotation. How much does the plumb bob deviate from a radial line at 35.0° north latitude? Assume that the Earth is spherical.

Section 6.4 Motion in the Presence of Resistive Forces

32. A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches a terminal speed of 50.0 m/s. (a) What is the acceleration of the sky diver when her speed is 30.0 m/s? What is the drag force on the diver when her speed is (b) 50.0 m/s? (c) 30.0 m/s?

33. A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by $a = g - bv$. After falling 0.500 m, the Styrofoam effectively reaches terminal speed, and then takes 5.00 s more to reach the ground. (a) What is the value of the constant b? (b) What is the acceleration at $t = 0$? (c) What is the acceleration when the speed is 0.150 m/s?

34. (a) Estimate the terminal speed of a wooden sphere (density 0.830 g/cm^3) falling through air if its radius is 8.00 cm and its drag coefficient is 0.500. (b) From what height would a freely falling object reach this speed in the absence of air resistance?

35. Calculate the force required to pull a copper ball of radius 2.00 cm upward through a fluid at the constant speed 9.00 cm/s. Take the drag force to be proportional to the speed, with proportionality constant 0.950 kg/s. Ignore the buoyant force.

36. A fire helicopter carries a 620-kg bucket at the end of a cable 20.0 m long as in Figure P6.36. As the helicopter flies to a fire at a constant speed of 40.0 m/s, the cable makes an angle of 40.0° with respect to the vertical. The bucket presents a cross-sectional area of 3.80 m^2 in a plane perpendicular to the air moving past it. Determine the drag coefficient assuming that the resistive force is proportional to the square of the bucket's speed.

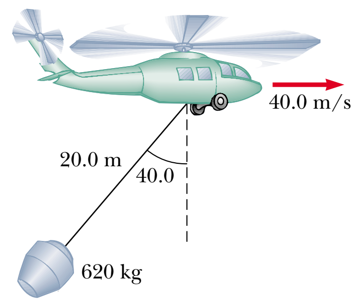


Figure P6.36

37. A small, spherical bead of mass 3.00 g is released from rest at $t = 0$ in a bottle of liquid shampoo. The terminal speed is observed to be $v_T = 2.00$ cm/s. Find (a) the value of the constant b in Equation 6.2, (b) the time τ at which the bead reaches $0.632v_T$, and (c) the value of the resistive force when the bead reaches terminal speed.

38. The mass of a sports car is 1 200 kg. The shape of the body is such that the aerodynamic drag coefficient is 0.250 and the frontal area is 2.20 m^2. Neglecting all other sources of friction, calculate the initial acceleration of the car if it has been traveling at 100 km/h and is now shifted into neutral and allowed to coast.

39. A motorboat cuts its engine when its speed is 10.0 m/s and coasts to rest. The equation describing the motion of the motorboat during this period is $v = v_i e^{-ct}$, where v is the speed at time t, v_i is the initial speed, and c is a constant. At $t = 20.0$ s, the speed is 5.00 m/s. (a) Find the constant c. (b) What is the speed at $t = 40.0$ s? (c) Differentiate the expression for $v(t)$ and thus show that the acceleration of the boat is proportional to the speed at any time.

40. Consider an object on which the net force is a resistive force proportional to the square of its speed. For example, assume that the resistive force acting on a speed skater is $f = -kmv^2$, where k is a constant and m is the skater's mass. The skater crosses the finish line of a straight-line race with

speed v_0 and then slows down by coasting on his skates. Show that the skater's speed at any time t after crossing the finish line is $v(t) = v_0/(1 + ktv_0)$. This problem also provides the background for the two following problems.

41. (a) Use the result of Problem 40 to find the position x as a function of time for an object of mass m, located at $x = 0$ and moving with velocity $v_0\hat{\mathbf{i}}$ at time $t = 0$ and thereafter experiencing a net force $-kmv^2\hat{\mathbf{i}}$. (b) Find the object's velocity as a function of position.

42. At major league baseball games it is commonplace to flash on the scoreboard a speed for each pitch. This speed is determined with a radar gun aimed by an operator positioned behind home plate. The gun uses the Doppler shift of microwaves reflected from the baseball, as we will study in Chapter 39. The gun determines the speed at some particular point on the baseball's path, depending on when the operator pulls the trigger. Because the ball is subject to a drag force due to air, it slows as it travels 18.3 m toward the plate. Use the result of Problem 41(b) to find how much its speed decreases. Suppose the ball leaves the pitcher's hand at 90.0 mi/h = 40.2 m/s. Ignore its vertical motion. Use data on baseballs from Example 6.13 to determine the speed of the pitch when it crosses the plate.

43. You can feel a force of air drag on your hand if you stretch your arm out of the open window of a speeding car. [*Note:* Do not endanger yourself.] What is the order of magnitude of this force? In your solution state the quantities you measure or estimate and their values.

Section 6.5 Numerical Modeling in Particle Dynamics

44. A 3.00-g leaf is dropped from a height of 2.00 m above the ground. Assume the net downward force exerted on the leaf is $F = mg - bv$, where the drag factor is $b = 0.030\,0$ kg/s. (a) Calculate the terminal speed of the leaf. (b) Use Euler's method of numerical analysis to find the speed and position of the leaf, as functions of time, from the instant it is released until 99% of terminal speed is reached. (*Suggestion:* Try $\Delta t = 0.005$ s.)

45. A hailstone of mass 4.80×10^{-4} kg falls through the air and experiences a net force given by

$$F = -mg + Cv^2$$

where $C = 2.50 \times 10^{-5}$ kg/m. (a) Calculate the terminal speed of the hailstone. (b) Use Euler's method of numerical analysis to find the speed and position of the hailstone at 0.2-s intervals, taking the initial speed to be zero. Continue the calculation until the hailstone reaches 99% of terminal speed.

46. A 0.142-kg baseball has a terminal speed of 42.5 m/s (95 mi/h). (a) If a baseball experiences a drag force of magnitude $R = Cv^2$, what is the value of the constant C? (b) What is the magnitude of the drag force when the speed of the baseball is 36.0 m/s? (c) Use a computer to determine the motion of a baseball thrown vertically upward at an initial speed of 36 m/s. What maximum height does the ball reach? How long is it in the air? What is its speed just before it hits the ground?

47. A 50.0-kg parachutist jumps from an airplane and falls to Earth with a drag force proportional to the square of the speed, $R = Cv^2$. Take $C = 0.200$ kg/m (with the parachute closed) and $C = 20.0$ kg/m (with the chute open). (a) Determine the terminal speed of the parachutist in both configurations, before and after the chute is opened. (b) Set up a numerical analysis of the motion and compute the speed and position as functions of time, assuming the jumper begins the descent at 1 000 m above the ground and is in free fall for 10.0 s before opening the parachute. (*Suggestion:* When the parachute opens, a sudden large acceleration takes place; a smaller time step may be necessary in this region.)

48. Consider a 10.0-kg projectile launched with an initial speed of 100 m/s, at an elevation angle of 35.0°. The resistive force is $\mathbf{R} = -b\mathbf{v}$, where $b = 10.0$ kg/s. (a) Use a numerical method to determine the horizontal and vertical coordinates of the projectile as functions of time. (b) What is the range of this projectile? (c) Determine the elevation angle that gives the maximum range for the projectile. (*Suggestion:* Adjust the elevation angle by trial and error to find the greatest range.)

49. A professional golfer hits her 5-iron 155 m (170 yd). A 46.0-g golf ball experiences a drag force of magnitude $R = Cv^2$, and has a terminal speed of 44.0 m/s. (a) Calculate the drag constant C for the golf ball. (b) Use a numerical method to calculate the trajectory of this shot. If the initial velocity of the ball makes an angle of 31.0° (the loft angle) with the horizontal, what initial speed must the ball have to reach the 155-m distance? (c) If this same golfer hits her 9-iron (47.0° loft) a distance of 119 m, what is the initial speed of the ball in this case? Discuss the differences in trajectories between the two shots.

Additional Problems

50. In a home laundry dryer, a cylindrical tub containing wet clothes is rotated steadily about a horizontal axis, as shown in Figure P6.50. So that the clothes will dry uniformly, they are made to tumble. The rate of rotation of the smooth-walled tub is chosen so that a small piece of cloth will lose contact with the tub when the cloth is at an angle of 68.0°

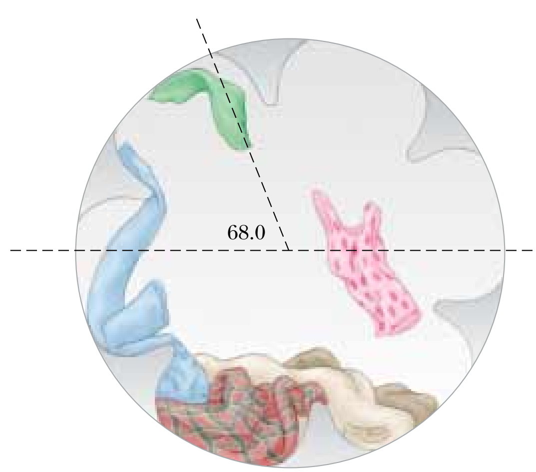


Figure P6.50

above the horizontal. If the radius of the tub is 0.330 m, what rate of revolution is needed?

51. We will study the most important work of Nobel laureate Arthur Compton in Chapter 40. Disturbed by speeding cars outside the physics building at Washington University in St. Louis, Compton designed a speed bump and had it installed. Suppose that a 1 800-kg car passes over a bump in a roadway that follows the arc of a circle of radius 20.4 m as in Figure P6.51. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at 30.0 km/h? (b) **What If?** What is the maximum speed the car can have as it passes this highest point without losing contact with the road?

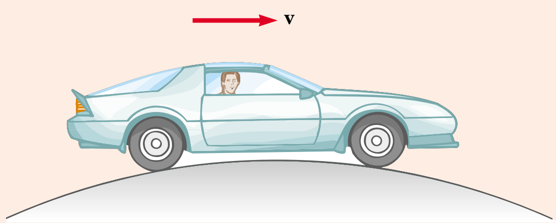


Figure P6.51 Problems 51 and 52.

52. A car of mass m passes over a bump in a road that follows the arc of a circle of radius R as in Figure P6.51. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at a speed v? (b) **What If?** What is the maximum speed the car can have as it passes this highest point without losing contact with the road?

53. Interpret the graph in Figure 6.18(b). Proceed as follows: (a) Find the slope of the straight line, including its units. (b) From Equation 6.6, $R = \frac{1}{2}D\rho Av^2$, identify the theoretical slope of a graph of resistive force versus squared speed. (c) Set the experimental and theoretical slopes equal to each other and proceed to calculate the drag coefficient of the filters. Use the value for the density of air listed on the book's endpapers. Model the cross-sectional area of the filters as that of a circle of radius 10.5 cm. (d) Arbitrarily choose the eighth data point on the graph and find its vertical separation from the line of best fit. Express this scatter as a percentage. (e) In a short paragraph state what the graph demonstrates and compare it to the theoretical prediction. You will need to make reference to the quantities plotted on the axes, to the shape of the graph line, to the data points, and to the results of parts (c) and (d).

54. A student builds and calibrates an accelerometer, which she uses to determine the speed of her car around a certain unbanked highway curve. The accelerometer is a plumb bob with a protractor that she attaches to the roof of her car. A friend riding in the car with her observes that the plumb bob hangs at an angle of 15.0° from the vertical when the car has a speed of 23.0 m/s. (a) What is the centripetal acceleration of the car rounding the curve? (b) What is the radius of the curve? (c) What is the speed of the car if the plumb bob deflection is 9.00° while rounding the same curve?

55. Suppose the boxcar of Figure 6.13 is moving with constant acceleration a up a hill that makes an angle ϕ with the horizontal. If the pendulum makes a constant angle θ with the perpendicular to the ceiling, what is a?

56. (a) A luggage carousel at an airport has the form of a section of a large cone, steadily rotating about its vertical axis. Its metallic surface slopes downward toward the outside, making an angle of 20.0° with the horizontal. A piece of luggage having mass 30.0 kg is placed on the carousel, 7.46 m from the axis of rotation. The travel bag goes around once in 38.0 s. Calculate the force of static friction between the bag and the carousel. (b) The drive motor is shifted to turn the carousel at a higher constant rate of rotation, and the piece of luggage is bumped to another position, 7.94 m from the axis of rotation. Now going around once in every 34.0 s, the bag is on the verge of slipping. Calculate the coefficient of static friction between the bag and the carousel.

57. Because the Earth rotates about its axis, a point on the equator experiences a centripetal acceleration of 0.033 7 m/s^2, while a point at the poles experiences no centripetal acceleration. (a) Show that at the equator the gravitational force on an object must exceed the normal force required to support the object. That is, show that the object's true weight exceeds its apparent weight. (b) What is the apparent weight at the equator and at the poles of a person having a mass of 75.0 kg? (Assume the Earth is a uniform sphere and take $g = 9.800$ m/s^2.)

58. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a counterweight of mass m_2 is tied to it (Fig. P6.58). The suspended object remains in equilibrium while the puck on the tabletop revolves. What is (a) the tension in the string? (b) the radial force acting on the puck? (c) the speed of the puck?

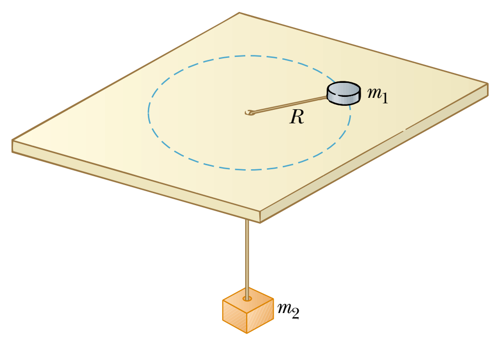


Figure P6.58

59. The pilot of an airplane executes a constant-speed loop-the-loop maneuver in a vertical circle. The speed of the airplane is 300 mi/h, and the radius of the circle is 1 200 ft. (a) What is the pilot's apparent weight at the lowest point if his true weight is 160 lb? (b) What is his apparent weight at the highest point? (c) **What If?** Describe how the pilot could experience weightlessness if both the radius and the speed can be varied. (*Note:* His apparent weight is equal to the magnitude of the force exerted by the seat on his body.)

60. A penny of mass 3.10 g rests on a small 20.0-g block supported by a spinning disk (Fig. P6.60). The coefficients of friction between block and disk are 0.750 (static) and

0.640 (kinetic) while those for the penny and block are 0.520 (static) and 0.450 (kinetic). What is the maximum rate of rotation in revolutions per minute that the disk can have, without the block or penny sliding on the disk?

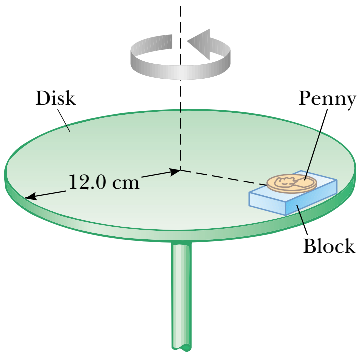


Figure P6.60

61. Figure P6.61 shows a Ferris wheel that rotates four times each minute. It carries each car around a circle of diameter 18.0 m. (a) What is the centripetal acceleration of a rider? What force does the seat exert on a 40.0-kg rider (b) at the lowest point of the ride and (c) at the highest point of the ride? (d) What force (magnitude and direction) does the seat exert on a rider when the rider is halfway between top and bottom?

Color Box/Getty Images

Figure P6.61

62. A space station, in the form of a wheel 120 m in diameter, rotates to provide an "artificial gravity" of 3.00 m/s^2 for persons who walk around on the inner wall of the outer rim. Find the rate of rotation of the wheel (in revolutions per minute) that will produce this effect.

63. An amusement park ride consists of a rotating circular platform 8.00 m in diameter from which 10.0-kg seats are suspended at the end of 2.50-m massless chains (Fig. P6.63). When the system rotates, the chains make an angle $\theta = 28.0°$ with the vertical. (a) What is the speed of each seat? (b) Draw a free-body diagram of a 40.0-kg child riding in a seat and find the tension in the chain.

64. A piece of putty is initially located at point A on the rim of a grinding wheel rotating about a horizontal axis. The putty is dislodged from point A when the diameter through A is horizontal. It then rises vertically and returns to A at the instant the wheel completes one revolution.

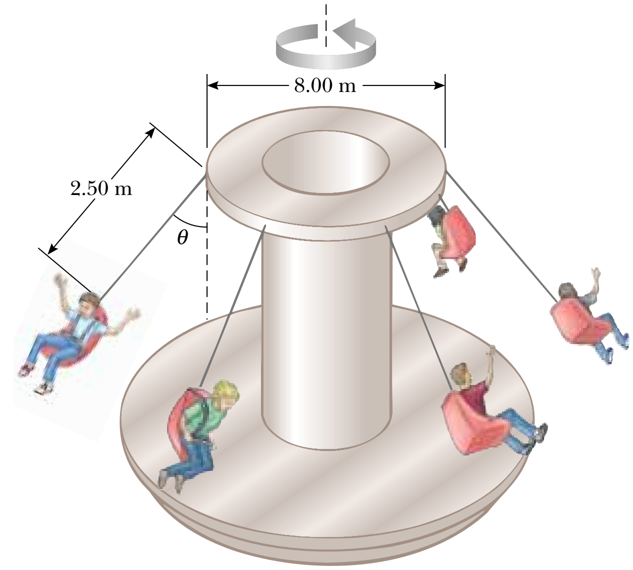


Figure P6.63

(a) Find the speed of a point on the rim of the wheel in terms of the acceleration due to gravity and the radius R of the wheel. (b) If the mass of the putty is m, what is the magnitude of the force that held it to the wheel?

65. An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough such that any person inside is held up against the wall when the floor drops away (Fig. P6.65). The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R. (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R\mu_s/g)^{1/2}$. (b) Obtain a numerical value for T if $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?

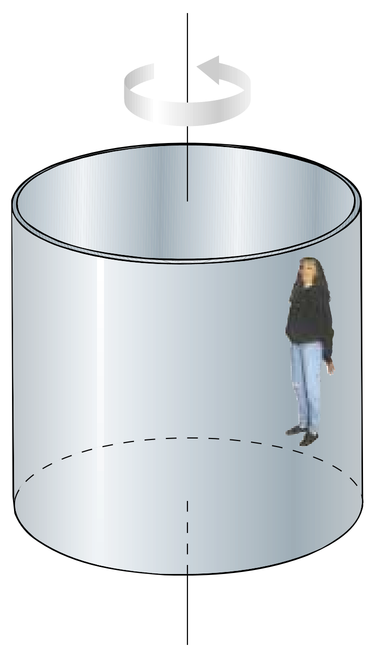


Figure P6.65

66. *An example of the Coriolis effect.* Suppose air resistance is negligible for a golf ball. A golfer tees off from a location precisely at $\phi_i = 35.0°$ north latitude. He hits the ball due south, with range 285 m. The ball's initial velocity is at 48.0° above the horizontal. (a) For how long is the ball in flight? The cup is due south of the golfer's location, and he would have a hole-in-one if the Earth were not rotating. The Earth's rotation makes the tee move in a circle of radius $R_E \cos \phi_i = (6.37 \times 10^6 \text{ m}) \cos 35.0°$, as shown in Figure P6.66. The tee completes one revolution each day. (b) Find the eastward speed of the tee, relative to the stars. The hole is also moving east, but it is 285 m farther south, and thus at a slightly lower latitude ϕ_f. Because the hole moves in a slightly larger circle, its speed must be greater than that of the tee. (c) By how much does the hole's speed exceed that of the tee? During the time the ball is in flight, it moves upward and downward as well as southward with the projectile motion you studied in Chapter 4, but it also moves eastward with the speed you found in part (b). The hole moves to the east at a faster speed, however, pulling ahead of the ball with the relative speed you found in part (c). (d) How far to the west of the hole does the ball land?

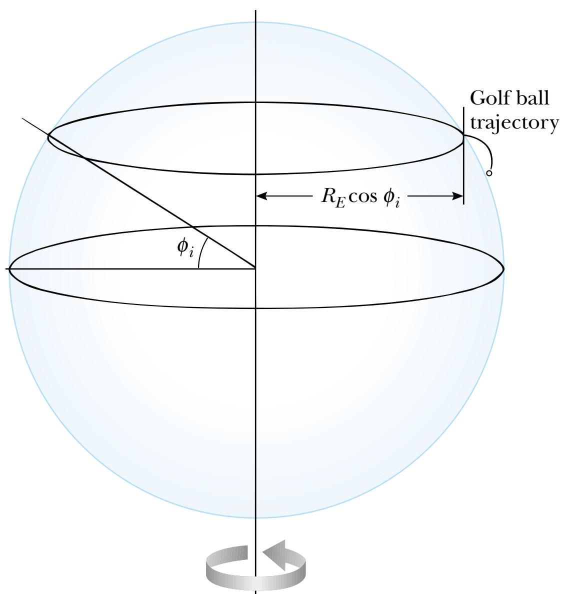


Figure P6.66

67. A car rounds a banked curve as in Figure 6.6. The radius of curvature of the road is R, the banking angle is θ, and the coefficient of static friction is μ_s. (a) Determine the range of speeds the car can have without slipping up or down the road. (b) Find the minimum value for μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10.0°$, and $\mu_s = 0.100$ (slippery conditions)?

68. A single bead can slide with negligible friction on a wire that is bent into a circular loop of radius 15.0 cm, as in Figure P6.68. The circle is always in a vertical plane and rotates steadily about its vertical diameter with (a) a period of 0.450 s. The position of the bead is described by the angle θ that the radial line, from the center of the loop to the bead, makes with the vertical. At what angle up from the bottom of the circle can the bead stay motionless relative to the turning circle? (b) **What If?** Repeat the problem if the period of the circle's rotation is 0.850 s.

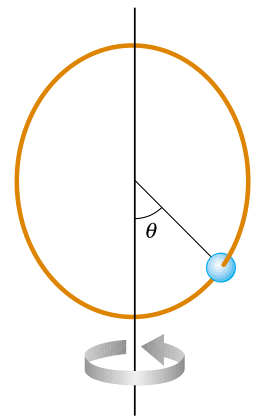


Figure P6.68

69. The expression $F = arv + br^2v^2$ gives the magnitude of the resistive force (in newtons) exerted on a sphere of radius r (in meters) by a stream of air moving at speed v (in meters per second), where a and b are constants with appropriate SI units. Their numerical values are $a = 3.10 \times 10^{-4}$ and $b = 0.870$. Using this expression, find the terminal speed for water droplets falling under their own weight in air, taking the following values for the drop radii: (a) 10.0 μm, (b) 100 μm, (c) 1.00 mm. Note that for (a) and (c) you can obtain accurate answers without solving a quadratic equation, by considering which of the two contributions to the air resistance is dominant and ignoring the lesser contribution.

70. A 9.00-kg object starting from rest falls through a viscous medium and experiences a resistive force $\mathbf{R} = -b\mathbf{v}$, where $\mathbf{v}$ is the velocity of the object. If the object reaches one-half its terminal speed in 5.54 s, (a) determine the terminal speed. (b) At what time is the speed of the object three-fourths the terminal speed? (c) How far has the object traveled in the first 5.54 s of motion?

$\boxed{71.}$ A model airplane of mass 0.750 kg flies in a horizontal circle at the end of a 60.0-m control wire, with a speed of 35.0 m/s. Compute the tension in the wire if it makes a constant angle of 20.0° with the horizontal. The forces exerted on the airplane are the pull of the control wire, the gravitational force, and aerodynamic lift, which acts at 20.0° inward from the vertical as shown in Figure P6.71.

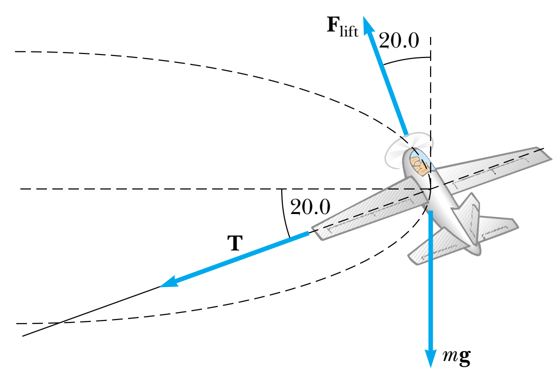


Figure P6.71

72. Members of a skydiving club were given the following data to use in planning their jumps. In the table, d is the distance fallen from rest by a sky diver in a "free-fall stable spread position," versus the time of fall t. (a) Convert the distances in feet into meters. (b) Graph d (in meters) versus t. (c) Determine the value of the terminal speed v_T by finding the slope of the straight portion of the curve. Use a least-squares fit to determine this slope.

t (s)	d (ft)	t (s)	d (ft)
1	16	11	1 309
2	62	12	1 483
3	138	13	1 657
4	242	14	1 831
5	366	15	2 005
6	504	16	2 179
7	652	17	2 353
8	808	18	2 527
9	971	19	2 701
10	1 138	20	2 875

73. If a single constant force acts on an object that moves on a straight line, the object's velocity is a linear function of time. The equation $v = v_i + at$ gives its velocity v as a function of time, where a is its constant acceleration. **What if** velocity is instead a linear function of position? Assume that as a particular object moves through a resistive medium, its speed decreases as described by the equation $v = v_i - kx$, where k is a constant coefficient and x is the position of the object. Find the law describing the total force acting on this object.

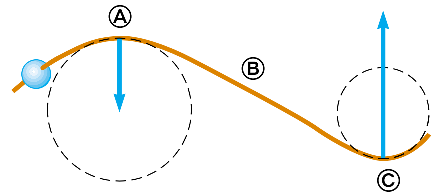
Answers to Quick Quizzes

6.1 (b), (d). The centripetal acceleration is always toward the center of the circular path.

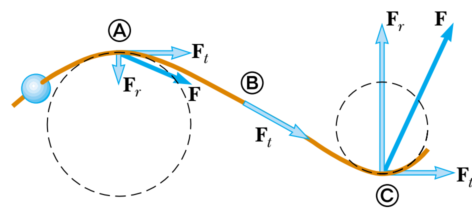
6.2 (a), (d). The normal force is always perpendicular to the surface that applies the force. Because your car maintains its orientation at all points on the ride, the normal force is always upward.

6.3 (a). If the car is moving in a circular path, it must have centripetal acceleration given by Equation 4.15.

6.4 Because the speed is constant, the only direction the force can have is that of the centripetal acceleration. The force is larger at Ⓒ than at Ⓐ because the radius at Ⓒ is smaller. There is no force at Ⓑ because the wire is straight.



6.5 In addition to the forces in the centripetal direction in Quick Quiz 6.4, there are now tangential forces to provide the tangential acceleration. The tangential force is the same at all three points because the tangential acceleration is constant.



6.6 (c). The only forces acting on the passenger are the contact force with the door and the friction force from the seat. Both of these are real forces and both act to the left in Figure 6.11. Fictitious forces should never be drawn in a force diagram.

6.7 (a). The basketball, having a larger cross-sectional area, will have a larger force due to air resistance than the baseball. This will result in a smaller net force in the downward direction and a smaller downward acceleration.